AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

UPS AND DOWNS is the theme
for this INCLUSION AND HOME LEARNING GUIDE
This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

The QUADRATIC MATCHING 1 activity was designed for Upper Secondary.

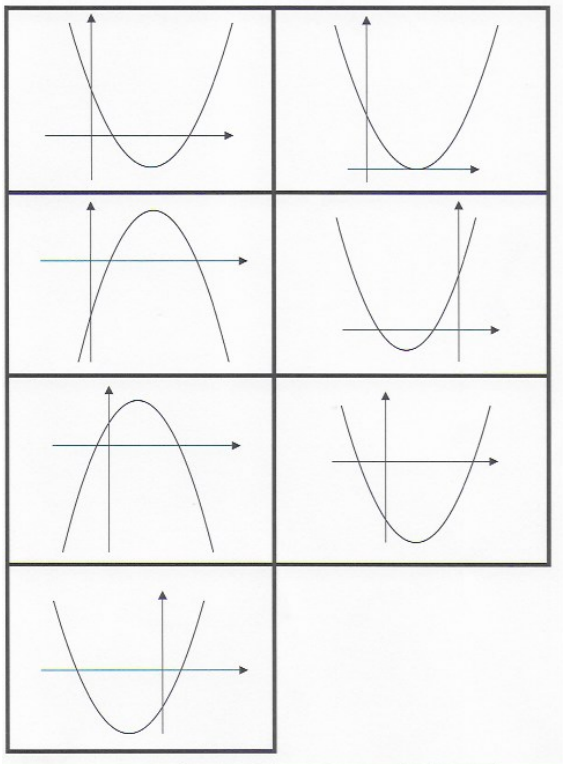
QUADRATIC MATCHING 1

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$

Graphs and equations of 7 quadratic functions are given here.

Match them and put them into 7 sets. Write down the coordinates of the intercepts with the axes.

Make a poster showing the graph of each function with the matching equations.



In all these examples the coefficient of the quadratic term is +1 or -1.

Choose your own quadratic function where this coefficient is not equal to +1 or -1 and complete your poster to include your own 8th set with its graph, equations and properties.

HELP

First use the cards in set C and match the equations of the quadratic functions with the factorised forms.

Then match the graphs given in set B to the equations to make up the 7 sets.

Then use the cards in set E and match this information about the intercepts of the graph with the axes to the 7 sets.

NEXT

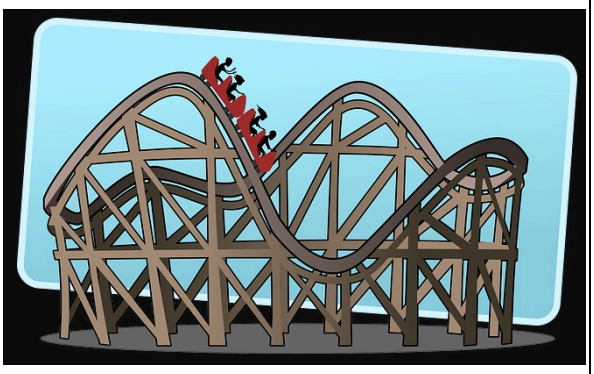
Match the remaining cards in sets A1 and A2 with the other cards.

Resources: Cards sets A1, A2, B, C, D and E.

INCLUSION AND HOME LEARNING GUIDE

THEME: UPS AND DOWNS

Early Years and Lower Primary



Talk about the UPS and DOWNS of rides on water slides, roller coasters, playground slides and skateboard ramps.

The children should look at the pictures and describe the rides from start to finish.

What are the similarities **in the shapes** of water slides and roller coasters?

What are the differences? Does a ride usually start at the top or at the bottom, and is this a difference between water slide and roller coaster rides? Why is that?

Is it exciting when a ride climbs steeply or drops steeply (suddenly)?

This discussion will help children to develop a vocabulary about curves that will be used later in describing graphs. It introduces ideas of steepness (gradient) which are part of everyday experience in walking up or down hills although they might not have experienced rides like this.

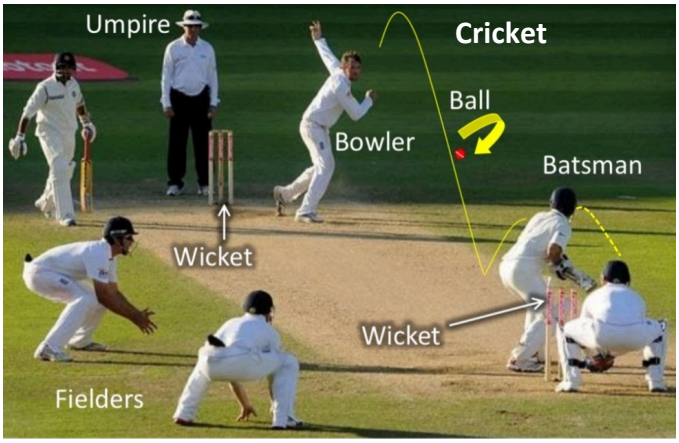
The water slides are actually in Muizenberg where the AIMSSEC office is located.

Upper Primary

17th century warfare. Cannon ball fired on a trajectory to go over a city wall and land in the city.

Describe the path of a ball when it is projected into the air either by throwing or by hitting it with a cricket bat or golf club.

Describe the path of a cannonball fired upwards so as to hit a lower target.



In this lesson, introduce ideas of the trajectory of an object thrown in the air, which is part of everyday experience. Go outside. Everyone should watch the path of a ball after it is thrown by one person and caught by another, and then describe what they have seen.

Discuss the paths (also called trajectories) shown in the pictures and discuss other similar examples.

Why is this subject important in sport?

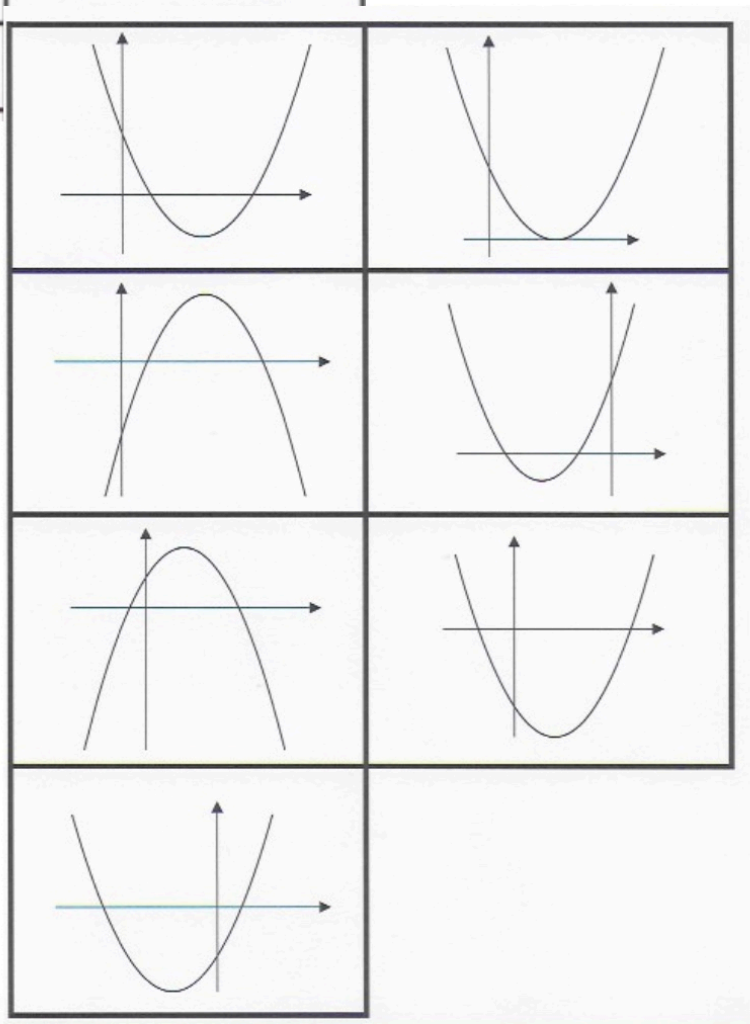
This discussion will help learners to develop a vocabulary and understanding about projectiles and their trajectories that will be useful later in physics and mathematics lessons. The shape of the path is called a parabola. The study of quadratic functions and their graphs (parabolic in shape) is important because it is part of the general theory of functions and their graphs, and it also has many applications to studying motion under gravity in physics, engineering and sport etc.

There is a horizontal component of velocity which does not change after the projectile starts on its path, unless affected by a strong wind perhaps, and also a vertical component of velocity that is affected by gravity and causes the projectile to move on a parabolic curve.

Lower Secondary

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	

Match the 7 equations to the seven graphs shown below.



For each equation, find the value of y when x = 0 and decide which graphs might have that equation and which are ruled out.

For each equation plot a few points that satisfy the equation, for example plot the points where x = -1, 0, 1, 2, 3, 4 and 5, and then match one of the sketch graphs with that equation.

If there are several learners in the group, each learner could work on a different equation. Then they could take it in turns to explain to the group what they have discovered.

This work does not require the learners to solve the equations, but if they know already how to solve quadratic equations, then they can use that knowledge to help them to make the matches.

Upper Secondary

Do the card matching task on page 1.

You have to sort the information into 7 sets of information with each set made up of a quadratic function written in 2 different forms and a graph. If possible work in a pair or small group and explain your reasoning to each other.

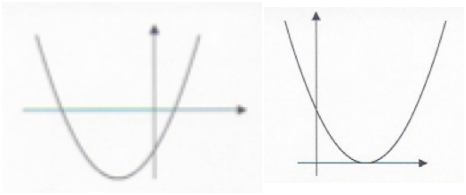
Once you have matched all seven, create one additional set of your own where the coefficient of the x^2 term is not +1 or -1.

Make a poster showing the 7 sets and the eighth one that you have created.

To review and extend your learning:

Take each set of cards in turn. Take turns to explain how you decided that a set of cards belong together.

Answer the questions:



Give me a possible equation for this graph.

Can you give me that equation in a different form?

Can you give me a completely different equation?

And ask all of the Key Questions below varying the coefficients.

Key questions

(1) What are the x and y intercepts of $y = (x - 8)(x + 2)$? How can you tell?

(2) What are the x and y intercepts of $y = (x - 4)^2$? How can you tell?

(3) Show me the equation of a quadratic that intercepts the y axis at -16. Now show me the same equation in a different form.

(4) Show me the equation of a quadratic that intersects the x axis at -4 and -2. Now show me the same equation in a different form.

In Physics or Applied maths lessons you may learn the theory behind the kinematics. In the following equation you will see that, if you know the distance s to the target, the horizontal component of initial velocity u, and the gravitational constant g, then you can calculate the time it takes for the projectile to hit the target.

DISTANCE TIME EQUATION FOR MOTION OF PROJECTILE.

$$s = ut - \frac{1}{2}gt^2$$

Notation:
Distance s; Time t; Horizontal component of initial velocity u; Gravitational constant g.

Why do this activity?

This activity provides a good review of quadratic functions. It can be done with sets of cards or from the worksheet on page 1.

The activity can be extended to work on the completed square form of quadratic equations and to finding maxima and minima by symmetry and inspection. See Quadratic Matching 2:

https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-2/

This leads to work on transformations of graphs and finding turning points by differentiation.

Learning objectives

In doing this activity students will have an opportunity to:

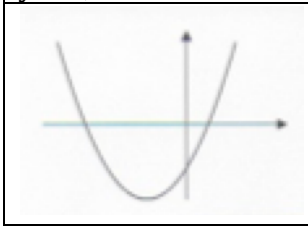
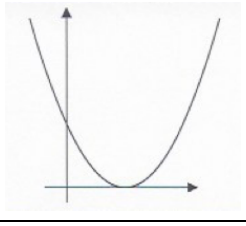
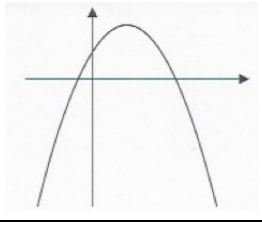
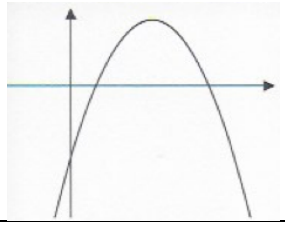
- identify different forms and properties of quadratic functions;
- connect quadratic functions with their graphs and properties including intersections with axes.

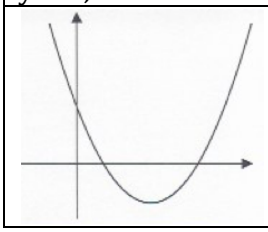
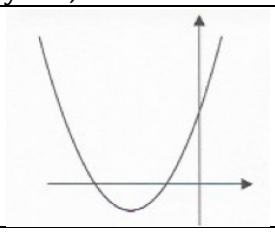
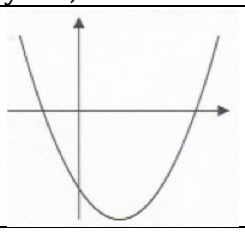
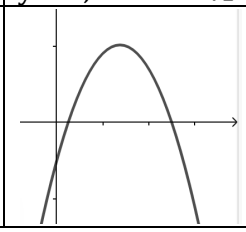
Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- develop the **skill of visualizing,** interpreting and/or creating visual images to represent concepts and situations.

SOLUTION

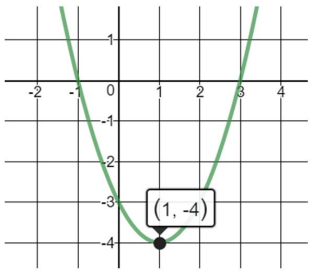
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$	$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = (x + 8)(x - 2)$	$y = (x - 4)(x - 4)$	$y = (x + 2)(4 - x)$	$y = (x - 2)(4 - x)$
$x = 0, y = -16$	$x = 0,\ y = 16$	$x = 0,\ y = 8$	$x = 0,\ y = -8$
$y = 0, x = -8$ or 2	$y = 0, x = 4$	$y = 0, x = -2$ or 4	$y = 0, x = 2$ or 4
			

			EXAMPLE
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$	$y = x^2 - 6x - 16$	$y = 11x - 5 - 2x^2$
$y = (x - 2)(x - 8)$	$y = (x + 2)(x + 4)$	$y = (x + 2)(x - 8)$	$y = (2x - 1)(5 - x)$
$x = 0, y = 16$	$x = 0,\ y = 8$	$x = 0,\ y = -16$	$x = 0, y = -5$
$y = 0, x = 2$ or 8	$y = 0, x = -2$ or -4	$y = 0, x = -2$ or 8	$y = 0, x = 5$ or $\frac{1}{2}$
			

DIAGNOSTIC ASSESSMENT This can be used before or after the lesson.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



Which of the following could be the equation of this curve?

A $y = (x + 1)^2 - 4$

B $y = (x - 1)^2 + 4$

C $y = (x + 1)^2 + 4$

D $y = (x - 1)^2 - 4$

1.Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.

2.It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.

3.With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.

4.Ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers.

Look **f**or a change an**d** who gave right and wrong answers. The correct answer is: D

Common Misconceptions

A. Most common mistake. Learner might know the turning point is (1,-4) and see +1 and -4 in the equation.

B. "I think this because the 4 in the equation has to be positive and the 1 has to be negative as the coordinates are opposite."

5. C. Probably has no idea and just guessed. https://diagnosticquestions.com

Follow up

Quadratic Matching 2:

https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-2/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel**

https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13

CARD SET A1 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties.
The quadratic functions are written in the forms: $y = ax^2 + bx + c$; $y = (x + p)(x + q)$; and $y = a(x + r)^2 + s$

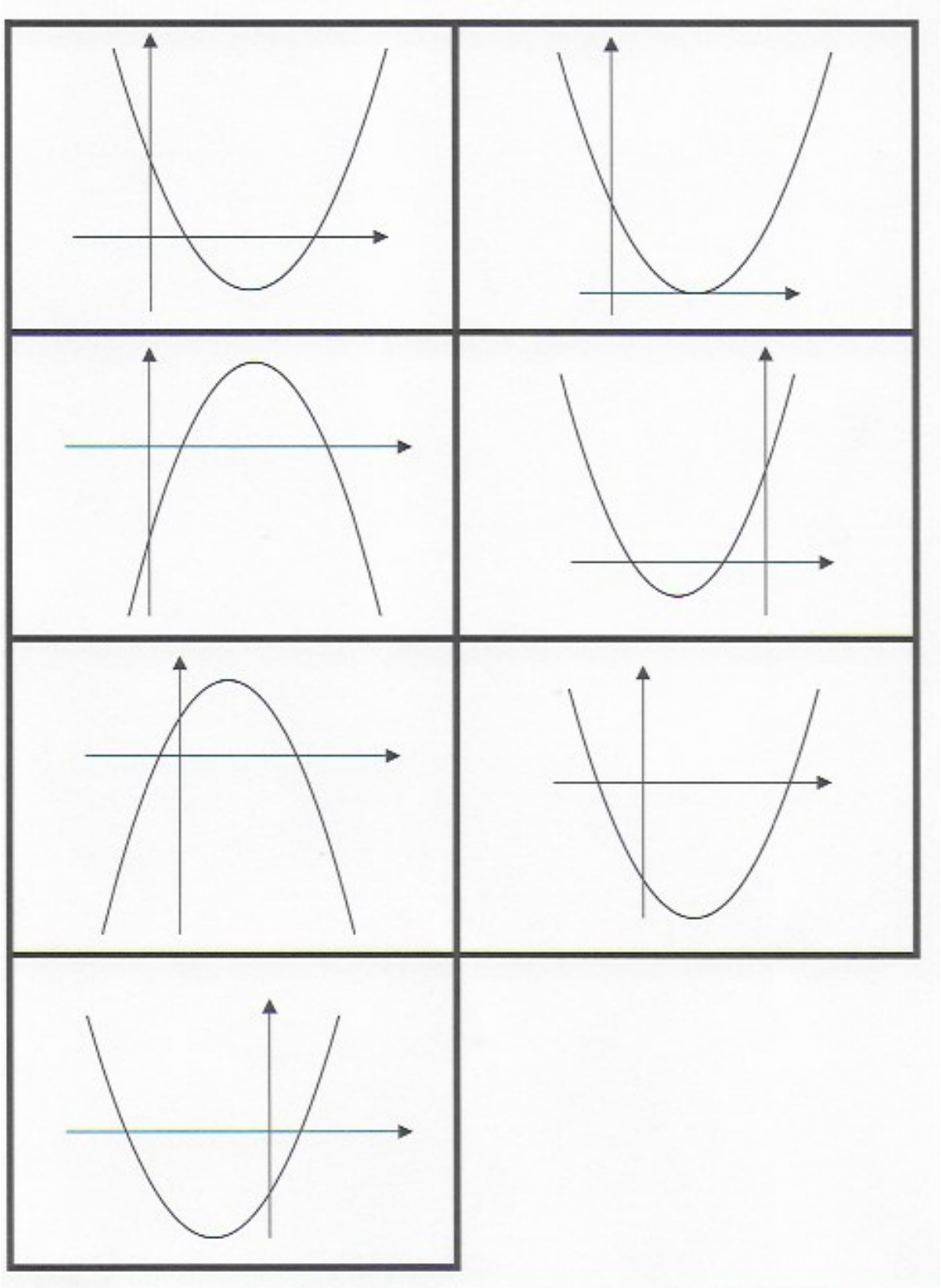
$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$
$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	**Minimum at (3, –25)**
Minimum at (–3, –1)	**Maximum at (1, 9)**

CARD SET A2 Sort the cards into 7 sets corresponding to 7 quadratic functions and their properties
The quadratic functions are written in the forms: $y = ax^2 + bx + c$; $y = (x + p)(x + q)$; and $y = a(x + r)^2 + s$

Maximum at (3, 1)	Minimum at (5, –9)
Minimum at (4, 0)	Minimum at (–3, –25)
$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2

CARD SET B

Match the graphs to the corresponding cards showing the equations and properties of the functions.



CARD SET C

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$

CARD SET D

$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	Minimum at (3, –25)
Minimum at (–3, –1)	Maximum at (1, 9)

CARD SET E Intercepts with the axes

$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2

Adapted from the STANDARDS UNIT professional development materials produced by the UK Department for Education and Skills. Author Malcolm Swan.