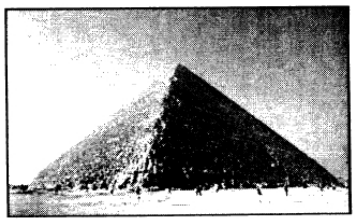
GREAT PYRAMID

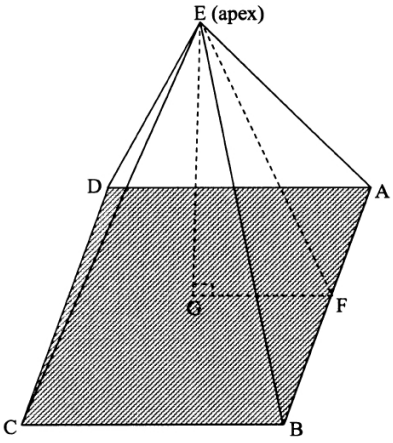


Great Pyramid at Giza in Egypt

The Great Pyramid at Giza in Egypt was built around 2500 BC. The pyramid has a square base ABCD with sides 232.6 metres long. The distance from each corner of the base to the apex E was originally 221.2 metres.

Draw a diagram of the pyramid to show two right angled triangles from which you can write down trigonometric equations to answer the following questions.

You may find it helpful to make a square based pyramid with rolled paper sticks and cut out a paper triangle like triangle EGF to fit inside.



1. If F is a point on AB such that EF is perpendicular to AB and G is vertically below the apex E, what can you say about triangles EFA and EGF?

2. Calculate the size of the angle at the apex of a face of the pyramid (for example angle AEB).
Can you find this by a different method?

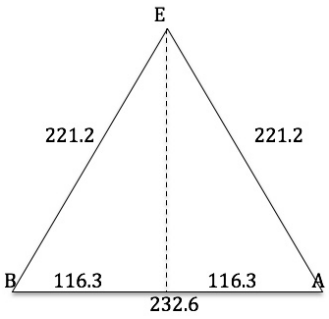
3. Calculate the angle each face makes with the base (for example angle EFG).

4. Calculate the height of the pyramid.

This question is from the South African National School Certificate Examination Mathematics Paper 2 November 2013 Qu. 13
Solution on page 4. Try to do the question without looking at the solution.

HELP

You will find it useful to draw this diagram so that it is easy to see the right-angled triangles.



NEXT

Find as many lengths and angles in the pyramid as possible, for example CG and angle ECG.

GUIDE FOR HOME LEARNING

Think about the future that lies ahead for young people. They need to be prepared for a job market where existing knowledge and skills have limited value unless they can be applied in new ways to solve today's complex problems and to improve the quality of life for all. To do well in life, we need to be confident that we can learn and apply new ideas. The quality of children's lives now, and as adults, is much more likely to be good if they have natural curiosity and enjoy learning, and if they like problem solving and independently finding things out for themselves. Everyone needs a GROWTH MINDSET which means knowing and having confidence that they can succeed in what they are interested in doing if they persevere and work hard.

Why do this activity?

This activity gives students an opportunity to develop their ability to visualise 3D structures and to apply the trigonometry they have learned to a real world problem. It is important for students to be able to use and apply the mathematics that they learn.

Learning objectives

In doing this activity students will have an opportunity to:

- develop visualisation of 3D structures;
- practise solving problems using right angled triangles and trigonometry.

Generic competences

In doing this activity students will have an opportunity to **develop visualization** and apply their knowledge to **solve problems** in a real life context.

Suggestions for Home Learning

PRIMARY AND LOWER SECONDARY

What can you find out about the Great Pyramid of Giza?

Make lots of paper sticks and make as many different types of pyramid as you can.

1. A square based pyramid.
2. A regular tetrahedron.
3. Pyramids with triangular, pentagonal, hexagonal and other polygonal bases.
4. Pyramids where the sloping faces are isosceles triangles.
5. A tetrahedron with 3 faces that are right-angled triangles.
6. A tetrahedron where the triangles are **no**t equilateral or isosceles or right-angled.

Describe the ***differences*** between **PYRAMIDS** and **PRISMS**.

Find some pyramids and/or prisms in your house.

Make some prisms.

1. A prism with a triangular cross section.
2. A prism with a square or rectangular cross section.
3. A prism with a circular cross section (use paper or card for this, not paper sticks).

UPPER SECONDARY

You may find it helpful to make a square based pyramid with paper sticks and to cut out a paper triangle like triangle EGF to fit inside.

Keep a supply of paper sticks so that you can quickly make models for different purposes. Just roll the paper tightly around a piece of string and secure with selotape. When you make models tie the strings at the vertices tightly but so that they can easily be undone. Untie the string and dismantle the model if you want ease of storage for your paper sticks.

It is important for everyone to be good at reading comprehension and this needs practice. Students need to read the question ***by themselves*** and draw a diagram of the pyramid and label the vertices, then to find two right-angled triangles. Label them EFA and EGF. If you find this difficult then do your best, and then work with someone else and talk to each other about what the question means. If at home, and you are not talking to a teacher or to a fellow student, then try to explain to whoever is helping you what you understand and what you are trying to work out.

After that write down trigonometric expressions to find angles and lengths in the diagram.

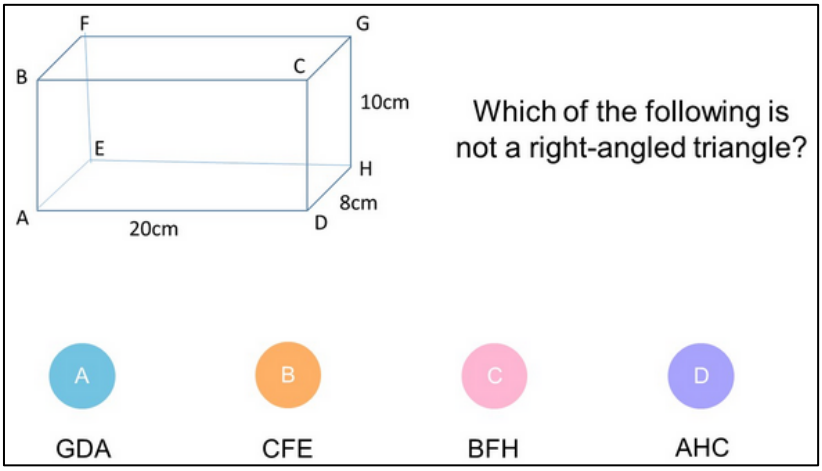
Answering the Key Questions will help you to solve the problem.

Key questions

- Can you draw (or show me) any right-angled triangles in the pyramid?
- What can you say about triangles EFA and EFB?
- What can you say about triangles EFG?
- What lengths do you know in that triangle?
- If you know two lengths in a right-angled triangle how can you find the angles?
- Can you use the sine rule or cosine rule?
- Can you use Pythagoras Theorem?

Diagnostic Assessment

You can test yourself. Answers on the next page.



https://diagnosticquestions.com

Follow up

In a soccer stadium https://aiminghigh.aimssec.ac.za/year-12-in-a-soccer-stadium/

SOLUTION

1. If F is a point on AB such that EF is perpendicular to AB and G is vertically below the apex E, then triangles EFA and EGF are right angled.

2. AF = 116.3 m (half the length of AB).
   EA = 221.2 m (the sloping edge length given)
   sin ∠AEF = 116.3/221.2 = 0.5258
   ∠AEF = 31.7°
   ∠AEB = 63.4°

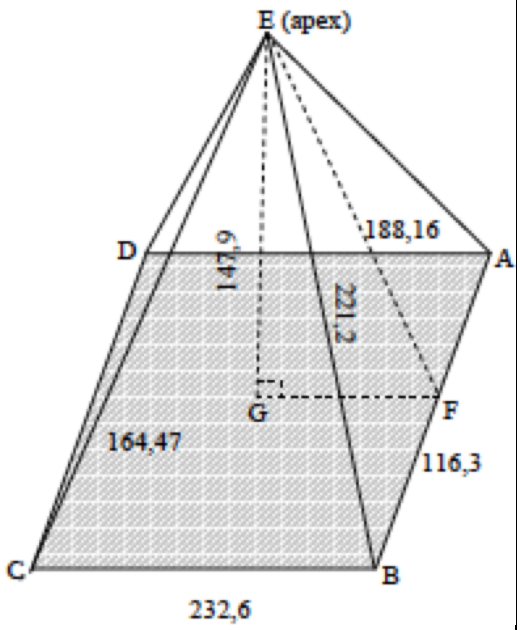
Alternative method using the cosine rule for ΔAEB:

$2(221.2)^2 \cos \angle AEB = 221.2^2 + 221.2^2 - 232.6^2$

$\cos \angle AEB = 1 - \frac{1}{2}(232.6)^2/(221.2)^2$

$= 0.4471$

∠AEB = 63.4°



3. By Pythagoras Theorem
   $EF^2 = 221.2^2 - 116.3^2 = 35403.75$
   EF = 188.16 m
   cos ∠EFG = 116.3/118.16 = 0.6181
   ∠EFG = 51.8°

4. EG = 116.3tan ∠EFG = 147.9m

DIAGNOSTIC QUIZ

D is the correct answer.

Common Misconceptions

A) AD⊥CDHG so AD⊥DG (line in plane CDHG) so GDA is a rt triangle. Learners may think the lines should go to opposite corners of the shape in order to be at right angles

B) EF⊥BCGF so EF⊥FC (line in plane BCGF) so CFE is a rt triangle.
Learners may think CFE is not a right-angled triangle because it does not look like one in the diagram.

C) BF⊥ EFGH so BF⊥FH (line in plane EFGH) so BFH is a rt triangle.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6