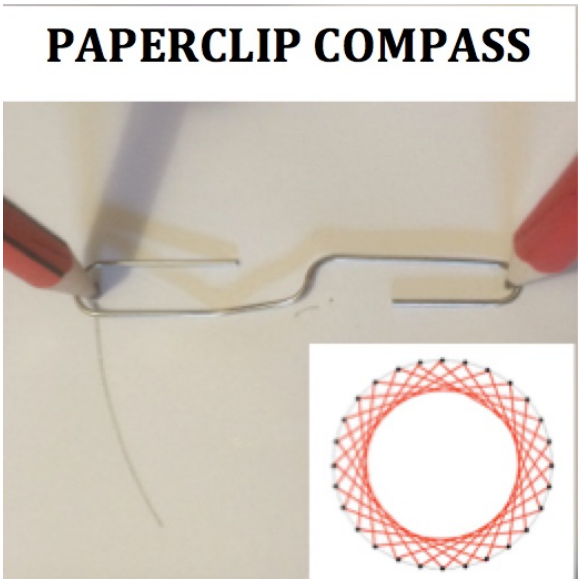
CONSTRUCT CIRCLE AND LINE PATTERNS



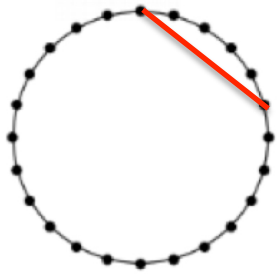
Double click on the picture to start the movie showing how to draw a circle using the paperclip compass.
Or go to: https://youtu.be/iewxclQEAnk

ENVELOPE PATTERNS WITH CIRCLES AND STRAIGHT LINES

Draw a circle and mark **24 points** equally spaced around the circumference. To do this, mark the centre, and mark your first point on the circumference.

With a protractor, starting from the radius from the centre to your 1st point, accurately measure an angle of 15^o at the centre of the circle and mark the second point on the circumference. Why 15^o?

Continue to measure angles of 15^o and to mark equally spaced points around the circle.



Then join each point to the 5th point around the circle missing 4 points between.

The inner circle that you see is called the **ENVELOPE** to the family of straight lines you have drawn. The lines are tangents to the envelope curve.

Name this envelope N=24, n→n + 5 where N is the number of points on the circle and n takes values 1, …, N.

What pattern would you get if you joined every 8th point on the 24 point circle, the N=24, n→n + 7 envelope?

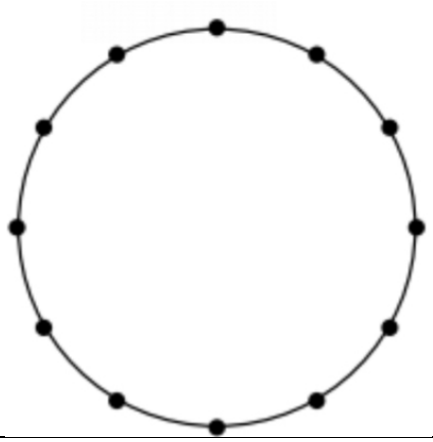
Experiment with different numbers of points around a circle, always joining points with straight lines and counting the same number of points between them.

The envelope shown above is formed by marking 28 points around a circle and joining each point to the 7th point around the circle (N=28, n→n + 7), missing 6 points between them. This is more difficult to draw accurately because the angle required is 360/28 = 12.86^o (to 2 decimal places).

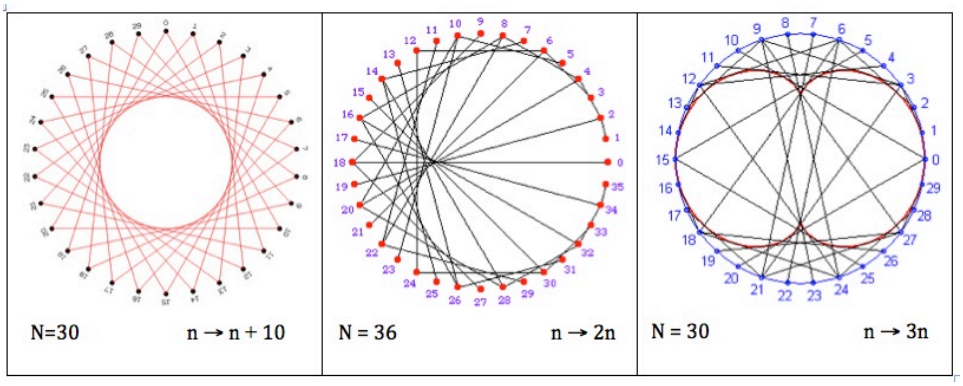
Help

Start by following the instructions for the circle with 12 points marked.
Join each point to the 5th point around the circle missing 4 points between.

Then use the circles with 6 points and 24 points.



Extension



Experiment with different mappings, for example
$n \rightarrow n + 10$,
$n \rightarrow 2n$
and $n \rightarrow 3n$
and make different patterns.

Each time try to find a connection between the mapping equation and the design.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

What is the order of rotational symmetry of this shape?

A	B	C	D
0	1	4	5

The correct answer is D

This is almost a ' you know it or you don't ' type of question but some students may recognize rotational symmetry but not know what the word 'order' means. Ask some of the students who got it right to explain to the class why the pattern has symmetry of order 5

This makes a useful lesson starter because it introduces the fundamental pattern making step that carries through the lesson. Start by marking points equally spaced around the circumference of a circle, then draw chords according to a given rule.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners practice in using a ruler, protractor and compasses to draw accurate geometrical constructions. To make the patterns learners will need to measure lengths and angles, to draw circles, to follow instructions and to draw accurately. The activity offers opportunities for talking about the geometrical properties of the shapes.

The activity could be developed into a class research project to investigate the connection between polygonal, circular, cardiod, nephroid and other envelopes and the pattern specification where N is the number of points on the circle and n→n + s gives s jumps between points.

Teachers can plan for learners of different abilities by giving learners different patterns to draw, for example slower learners could be given a circle with points already marked on the circumference to start with. The activity may improve learners' attitude to mathematics by appealing to some learners who do not like mathematics and to others who find it difficult. The activity encourages creativity. It offers opportunities for talking about the geometrical properties of the shapes and also for thinking mathematically about the classification of the patterns.

Intended learning outcomes

MEASURING ANGLES: Accurately use a protractor to measure and classify angles: $< 90^{o}$ (acute angles); Right-angles; angles $> 90^{o}$ (obtuse angles); Straight angles; $> 180^{o}$ (reflex angles).

CONSTRUCTIONS: Use a compass, ruler and protractor appropriately to construct geometric figures accurately, including: angles, to one degree of accuracy and circles.

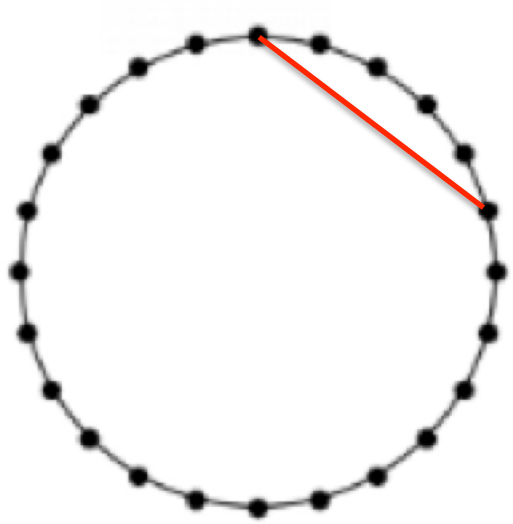
Generic Competences

This activity requires learners to:

- develop the skill of interpreting and creating visual images to represent concepts and situations;
- be creative and innovative - to apply knowledge and skills;
- be able to communicate in writing and speaking
    - co-operate - to collaborate/work in a team
    - to communicate, exchange ideas, criticise, and present information and ideas
    - to analyze, reason and communicate effectively.

Suggestions for teaching

Resources needed. Pencils, rulers, protractors and compasses.



Pin up a large copy of the 24-point circle.

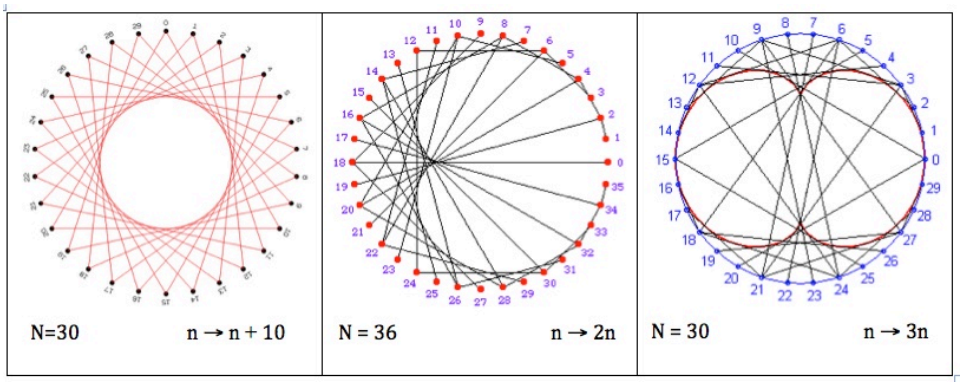
Explain that the class will help to make a pattern by drawing straight lines to join every 5th point on the circle. Ask learners to say what sort of pattern they think it will be.

Ask learners in turn to come to the front and each one to draw the next line.

Did they expect to see another circle appear?

Then tell the class that you want them to draw the pattern but they must first draw a circle and then mark 24 points equally spaced around the circle. Ask for suggestions about how they could do that and write the instructions on the board.

Circle **Cardioid** **Nephroid**



You could show this picture to the class (see page 7) and organise a class project. Assign a different pattern specification to each student. Then put their designs on display on the classroom wall.

Ask the students compare the patterns and explore possible rules that might relate to the type of envelope.

You might plan to cater for all abilities in your class by giving the slow learners a circle with the 24 points already marked and giving other learners circles with 6 or 12 of the 24 points marked but not all of the points, and asking the high flyers to mark all the points themselves.

Key questions

- What pattern would you get if you joined every 8th point on the 24 point circle?
- What pattern would you get if you joined every 6th point on the 24 point circle?
- What pattern would you get if you joined every 4th point on the 24 point circle?
- What pattern would you get if you joined every 3th point on the 24 point circle?

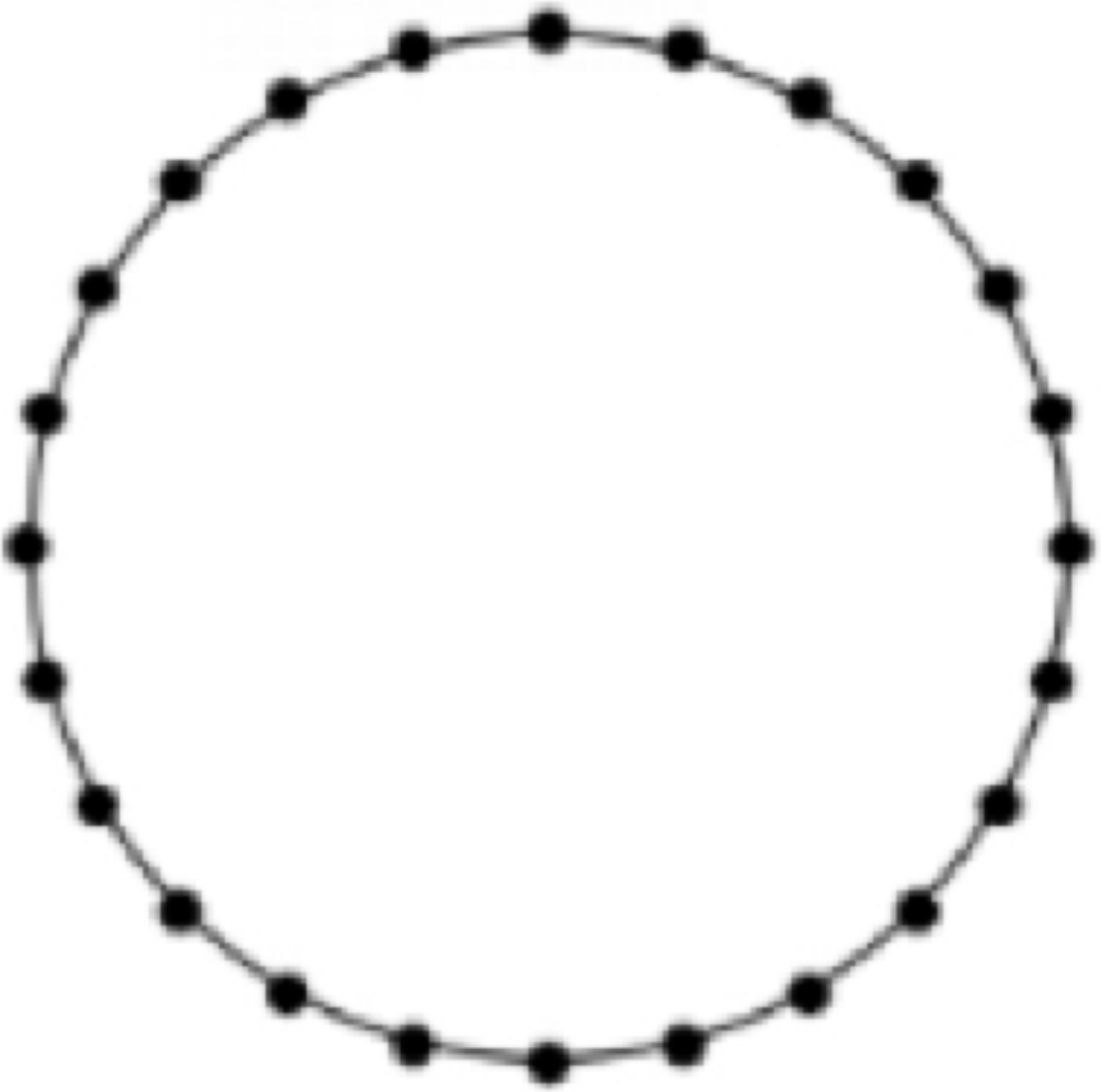
- What angle would you use to mark 30 points around the circle?
- What points would you join on the 30 point circle to get a hexagon?
- What points would you join on the 30 point circle to get a pentagon?
- Could you make a square by joining equally spaced points on a 30 point circle? Why or why not?
- What pattern would you get if you joined every 9^{th} point on the 30 point circle?
- Does your pattern have any symmetry?
- **And looking at a display of many patterns drawn by the members of the class:**
- Do you notice anything about N and s for the patterns that give polygons?
- Do you notice anything about N and s for the patterns that give circle envelopes?
- Do you notice anything about rules for drawing the patterns that give cardioid envelopes?
- Do you notice anything about the rules for drawing the patterns that give nephroid envelopes?
- Do you notice anything about the rules for drawing the patterns that give other envelopes?

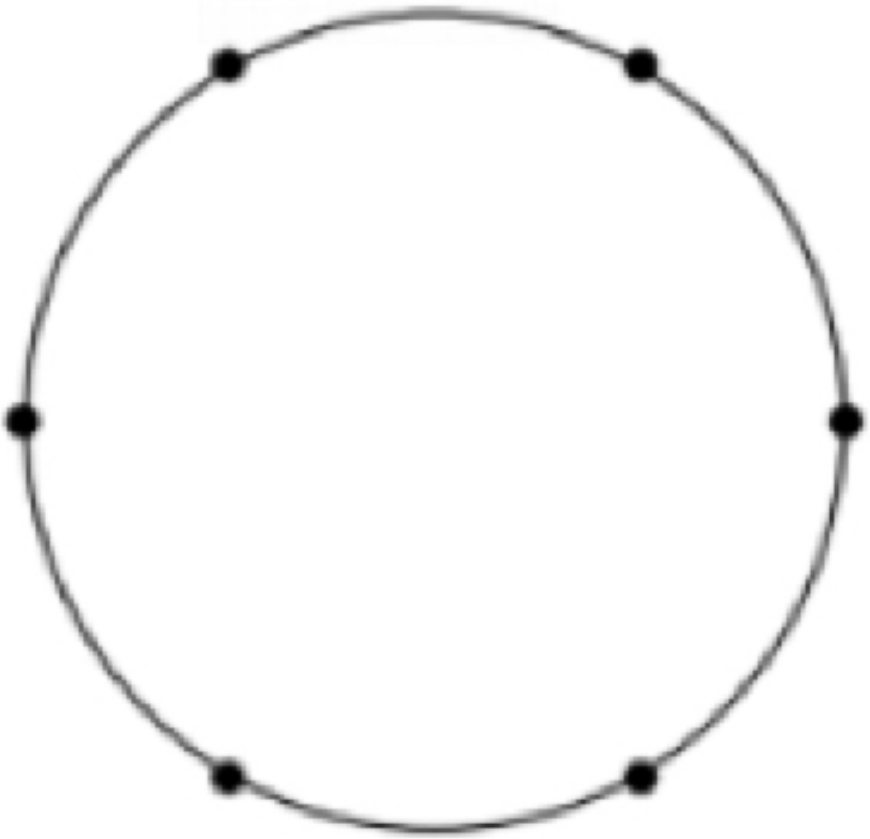
Follow up

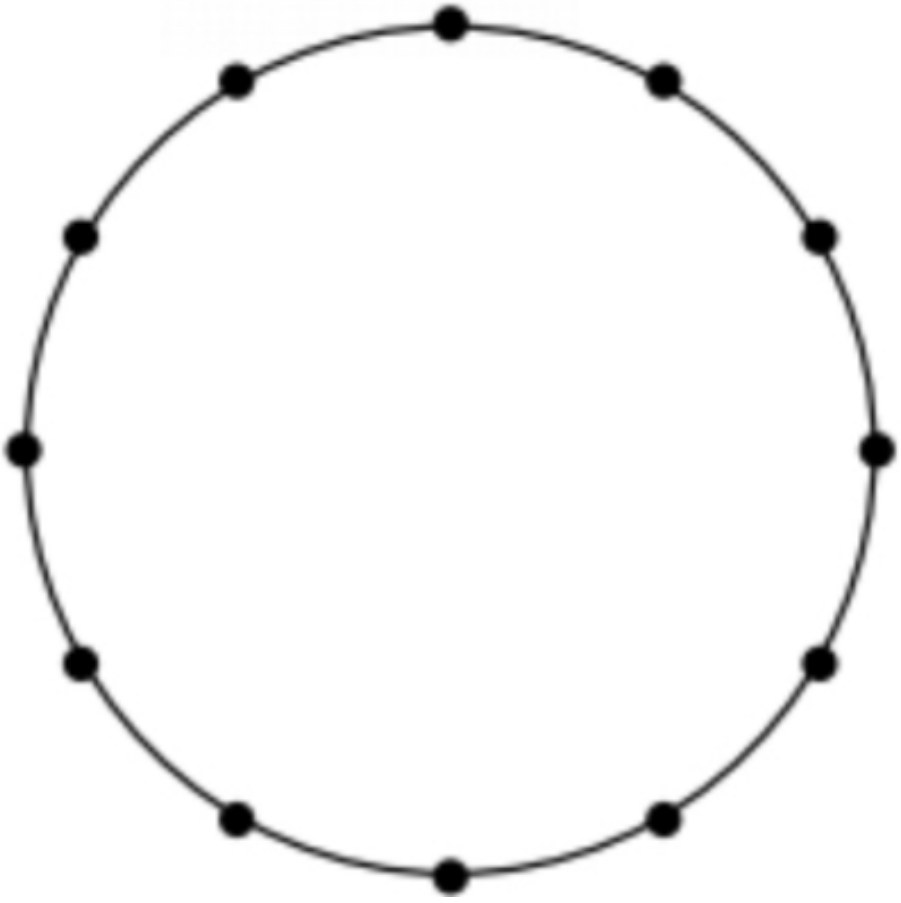
Clock Arithmetic and Envelopes:
https://aiminghigh.aimssec.ac.za/years-10-12-clock-arithmetic-and-envelopes/

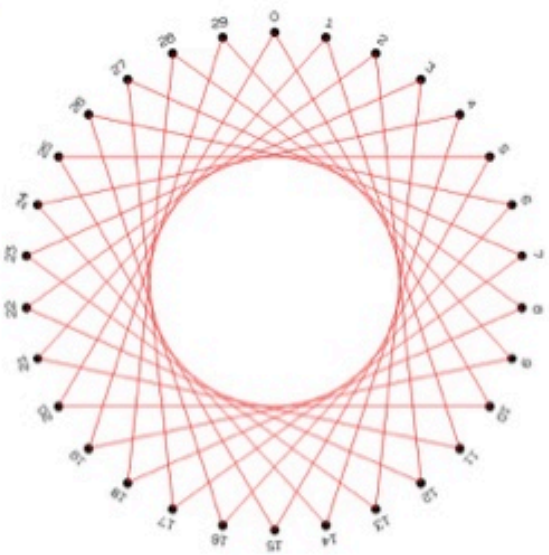
Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**

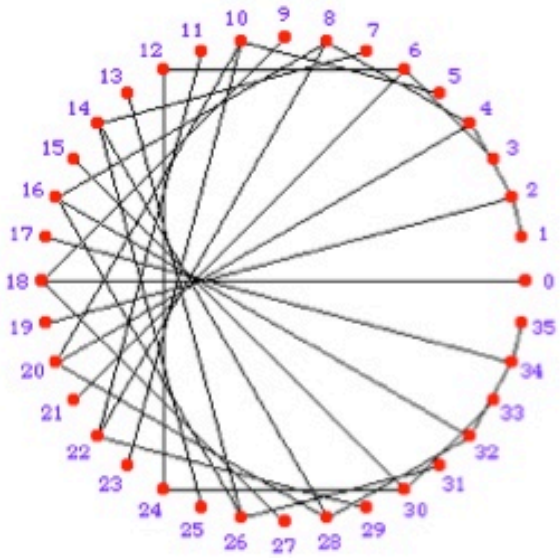




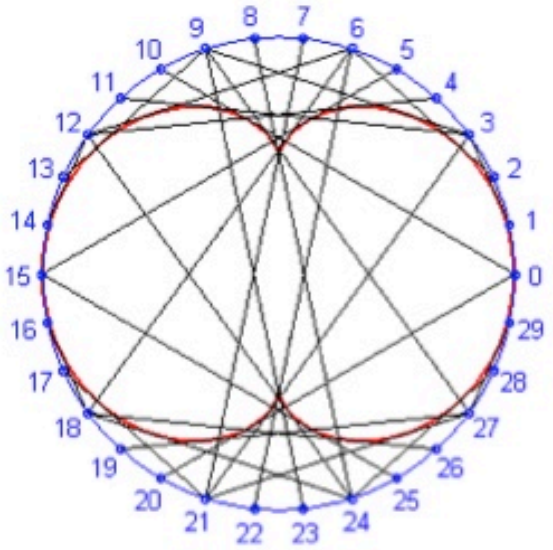




N=30 $n \to n + 10$



N = 36 $n \to 2n$



N = 30 $n \to 3n$