MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they are required to teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 **TITLE PAGE**
Teaching strategy.
Curriculum content and learning outcomes.
Summary of mathematical topic (FACT BOX.)

PAGES 2 & 3 **WORKSHOP ACTIVITIES FOR TEACHERS**
Two pages for you to work through with your colleagues. These are activities to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. pairs, whole group) and about how long the activity will take.

PAGES 4 & 5 **CLASSROOM ACTIVITIES FOR LEARNERS**
Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 TO 10 **CHANGES IN MY CLASSROOM PRACTICE**
Pages on implementing the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates.

Number Games

Teaching strategy: People maths, Visual and practical learning styles

Curriculum content: Development of number sense: patterns and relationships of numbers.

Prior knowledge needed: Secure counting skills and some knowledge of multiplication facts.

Intended Learning Outcomes: At the end of this activity teachers and learners will:

- Know how numbers are linked together in precise but different patterns and relationships;
- Understand these patterns and relationships follow certain rules and that these can be used and applied beyond the examples used;
- Be able to predict missing numbers and continue sequences where appropriate. Make reasoned judgements and use these to plan next steps;
- Appreciate that mathematics is a series of linked patterns and rules which can be used and applied in a variety of contexts;
- Have experienced mathematics learning in a fun and interactive way, Problem solving, Hypothesising and testing.

Fact box

A factor of a number is a number that can be divided into it exactly. 5 is a factor of 15.

A multiple of a number is made from adding the number to itself over and over again. So the multiples of 7 are: 7, 14, 21, 28....

Two numbers have a common factor when the same number is a factor of both. For example 6 and 15 have a common factor of 3.

Numbers which are multiples of *both* 3 *and* 5 are multiples of 15.
Another way of saying this is numbers which are in both the 3 and the 5 times table are also in the 15 times table.
Any number in the 15 times table has both 3 and 5 as factors.

We can say this about any two numbers which do not have a common factor;
for example:
2 and 3 do not have a common factor so any multiple of 6 has both 2 and 3 as factors,
4 and 5 do not have a common factor so any multiple of 20 has both 4 and 5 as factors,
3 and 8 do not have a common factor so any multiple of 24 has both 3 and 8 as factors.

Resources:
1 – 100 grids for each participant or squared paper.
Beans

Workshop Activities for Teachers

Activity 1: Counting Patterns

Whole group 30 minutes

The Pattern Game
All start sitting down.
All count in ones and emphasise the multiples of 3:
"1 … 2 … **3** … 4 … 5 … **6** … 7 … 8 … **9** … 10 … 11 … **12** …"
All stand every time a multiple of 3 is spoken and sit for the other numbers.

Repeat this game until everyone feels secure with this counting. For a challenge try counting back from 40 standing for each multiple of 3.

Now repeat but stand each time a multiple of 5 is spoken:

This time count:
"1 … 2 … 3 … 4 … **5**(stand) … 6 … 7 … 8 … 9 … **10**(stand) … 11 … 12 … 13 … 14 … **15** (stand) …" All stand for every multiple of 5

The 2 pattern game

Now the fun begins!!
Identify half the group to stand when multiples of 3 are spoken and the other half stand when multiples of 5 are spoken.
"1 … 2 … **3**(half stand) … 4 … **5** (other half stand)… **6** (first half stand)… 7 … 8 … **9** (first half stand)… **10** (other half stand)… 11 … **12** (first half stand) … 13 … 14 …**15** (all stand) …"
When everyone stands these numbers are multiples of both 3 and 5.

Notes **Topic:** Multiples

Start slowly so that no one is left behind. It may be helpful to have the counting numbers written on a board or use the one hundred squares at the end of this chapter for teachers to see. The layout on the one hundred square will help identify number patterns, particularly multiples of 5.
After each game, ask:
What are the first numbers called? (They are the multiples of 3. They all have a factor of 3.)
What are the second numbers called? (They are the multiples of 5. They all have a factor of 5.)
The game can be played again with different numbers. Easier numbers could be half stand for every multiple of 3 and others stand for every multiple of 10.

Activity 2: *Grab it*

Work with a partner. Grab a handful of beans. The aim of this game is to make as many different rectangular arrays as you can from your handful of beans. Putting all the beans in one line does not count as a rectangular array.
For each array you can make (and that your partner agrees with), score one point.
Keep a record of the numbers you have grabbed and the rectangles you have made.
Some numbers give you lots of points. They are the numbers with lots of factors, such as 24 and 30.
Prime numbers can't be made into any rectangular array, so you lose your go.

Activity 3: Patterns on 100 grid

Each teacher will need a 1-100 grids from the photo-copiable sheet in this chapter. If this is not possible, squared paper and one large grid for everyone to see.

If squared paper is being used fill in the numbers 1 to 100.
Circle the number 2. Put a line through every multiple of 2 up to 100.
Is there a pattern? What do you SEE?

Circle the number 3. Put a line through every multiple of 3 up to 100.

What do you notice about multiples of 4?

Circle the number 5. Put a line through all the multiples of 5 up to 100.

What do you notice about multiples of 6?
Circle the number 7. Put a line through every multiple of 7 up to 100.
What do you notice about the multiples of 8, 9 and 10?

What can you say about the numbers that aren't crossed out?

You might wonder why, unlike the patterns activity, you don't cross out the first number of a table. That's because the numbers that aren't crossed out are the prime numbers – those which have no factors other than themselves and 1. The first numbers of some tables (2,3,5,7) are prime.

There are lots of patterns to look for – numbers crossed out twice have two prime factors, numbers crossed out three times have three prime factors etc.

A Greek mathematician called Eratosthenes invented this systematic method of finding prime numbers. It is known as the Sieve of Eratosthenes. You could try extending this to higher numbers.

Notes to help you do Activity
Colours can be used

Discussion of pedagogical issues:

All three activities emphasise the same idea that some numbers are prime, some have few factors and others have lots. By doing different kinds of activities - those that work with sight, hearing and movement - around the same learning outcome we are offering all learners an opportunity to learn in the way which is most natural to them.

Classroom Activities for Learners

Activity 1: Number Game

Whole class 30 minutes

The Pattern Game
All start sitting down.
All count in ones and emphasise the multiples of 3:
"1 … 2 … **3** … 4 … 5 … **6** … 7 … 8 … **9** … 10 … 11 … **12** …"
All stand every time a multiple of 3 is spoken and sit for the other numbers.

Repeat this game until everyone feels secure with this counting. For a challenge try counting back from 40 standing for each multiple of 3.

Now repeat but stand each time a multiple of 5 is spoken:

This time count:
"1 … 2 … 3 … 4 … **5**(stand) … 6 … 7 … 8 … 9 … **10**(stand) … 11 … 12 … 13 … 14 … **15** (stand) …" All stand for every multiple of 5

The 2 pattern game

Now the fun begins!!
Identify half the group to stand when multiples of 3 are spoken and the other half stand when multiples of 5 are spoken.
"1 … 2 … **3**(half stand) … 4 … **5** (other half stand)… **6** (first half stand)… 7 … 8 … **9** (first half stand)… **10** (other half stand)… 11 … **12** (first half stand) … 13 … 14 …**15** (all stand) …"
When everyone stands these numbers are multiples of both 3 and 5.

Notes **Topic:** Multiples

Start slowly so that no one is left behind. Sitting and standing is good fun but learners can remain seated and clap instead of standing.

It may be helpful to have the one hundred square for learners to see. The layout will help identify number patterns, particularly multiples of 5.
After each game, ask:
What are the first numbers called? (They are the multiples of 3, and they all have a factor of 3.)
What are the second numbers called? (They are the multiples of 5, and they all have a factor of 5.)
When everyone stands these numbers are multiples of both 3 and 5.

Ideas for Teaching
These are very simple games for some learners but cause more difficulties for others. They are useful for learners beginning their multiplication tables and for second language learners to practice saying the numbers in English. Actively involving the learners in standing will make the learning fun
Can the pupils tell you in advance which will be multiples of 3? Can they point to them on a one hundred square?
You don't have to start counting at 1. You could start at 10: 10 … 11 … **12**…
Or you could start at 20 and count backwards: 20 … 19 … **18** … 17 … 16 …

The 2 pattern game will always cause a laugh as it becomes complicated focussing on both numbers at the same time. Talk about the number patterns using the words multiple and factor. Get learners to point to them on a one hundred square. Choose different numbers to help identify different patterns. For example, multiples of 3 and multiples of 6 would mean all stand for every multiple of 6 . Reinforcing that all multiples of 6 must also be multiples of 3.

Activity 3: *Patterns on 100 grid*

Each learner will need a 1-100 grids from the photo-copiable sheet in this chapter. If this is not possible, squared paper and one large grid for everyone to see.

If squared paper is being used fill in the numbers 1 to 100.
Circle the number 2. Put a line through every multiple of 2 up to 100.
Is there a pattern? What do you SEE?

Circle the number 3. Put a line through every multiple of 3 up to 100.

What do you notice about multiples of 4?

Circle the number 5. Put a line through all the multiples of 5 up to 100.

What do you notice about multiples of 6?
Circle the number 7. Put a line through every multiple of 7 up to 100.
What do you notice about the multiples of 8, 9 and 10?

What can you say about the numbers that aren't crossed out?

You might wonder why, unlike the patterns activity, you don't cross out the first number of a table. That's because the numbers that aren't crossed out are the prime numbers – those which have no factors other than themselves and 1. The first numbers of some tables (2,3,5,7) are prime.

There are lots of patterns to look for – numbers crossed out twice have two prime factors, numbers crossed out three times have three prime factors etc.

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Changes in my classroom

Mental activities

Starting each lesson with mental activities will make the learners alert and ready for connecting new ideas to the knowledge they already have.

Counting patterns will give the learners a feel for numbers as some crop up frequently, such as 12, while others do not appear, such as 13. Counting also goes beyond the times tables so that learners see numbers such as 48 have a factor 3.

Pedagogical Issues

Familiarity with patterns comes through oral and visual experiences. Involving learners actively in standing and sitting will make the learning fun and more memorable.

Helping learners remember

Making resources: Photocopy the one hundred square on the following pages and use a laminator to make a more durable classroom resource. A marker pen can be used to circle particular numbers, for example, multiples of 3. A large one hundred square displayed in the classroom will help learners see patterns of numbers.

Making a factor bug

First draw a body and a head for the bug

Write a number in the body.

For each factor draw a leg and the other leg in the factor pair.

Work through the numbers 2, 3, 4, 5, ….systematically so that no factors are missed.

Finally draw antennae with the number itself and the number 1 as the factor pair.

Any bug with no legs and only antennae will be a prime number.

Follow up activities

You could read more about the structure of factors and multiples in the article Using Arrays to explore Numbers http://nrich.maths.org/2466

For more activities on multiples and factors see Factor Track http://nrich.maths.org/7468 or Counting Cogs http://nrich.maths.org/6966 or Table Patterns Go Wild http://nrich.maths.org/6924

For similar activities to What's my number see I Like http://nrich.maths.org/6962 and Which Numbers 1 and 2 http://nrich.maths.org/6948 and http://nrich.maths.org/6949

One hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100