WORKHOP GUIDES FOR TEACHERS TO LEARN TOGETHER

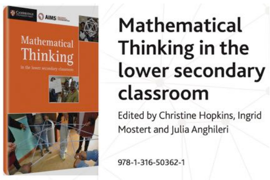
UPPER SECONDARY P1 CONTINGENCY TABLES & VENN DIAGRAMS

Guide for your own self-help professional development workshop
and resources for inquiry based lessons.

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



The AIMSSEC Mathematical Thinking Book provides similar workshop guides.
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EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 **TITLE PAGE**
Teaching strategy.
Curriculum content and learning outcomes.
Summary of mathematical topic (FACT BOX.)

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS
Two pages for you to work through with your colleagues. These are activities to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to pause, think and try the activity and when to record your work.

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS
Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 TO 10 **CHANGES IN MY CLASSROOM PRACTICE**
Pages on implementing the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates.

Contingency Tables and Venn Diagrams by Judy Paterson 1948 – 2015

Judy was involved from the start in the planning for AIMSSEC and taught the first AIMSSEC course in Stellenbosch in 2003. She was born in Zambia and moved to South Africa as a child. She taught at Leif College, a school for historically disadvantaged students in South Africa. She moved to Auckland in 1994 after her husband died where she led an innovative mathematics teacher training programme. Her PhD explored her hypothesis that teachers could be led to reconsider their pedagogy by inspiring them with new mathematics content learning. The Judy Paterson Scholarship was established in 2014 in recognition of her contribution to Mathematics Education, and especially her work with secondary school mathematics teachers in Auckland.

Contingency Tables and Venn Diagrams

Teaching strategy: Developing Mathematical Language & Understanding

Curriculum content: Contingency tables and Venn diagrams as aids to solving probability problems (where events are not necessarily independent).

Prior knowledge needed: Operations with fractions, decimals and percentages; knowledge of probability as the fraction of successful outcomes.

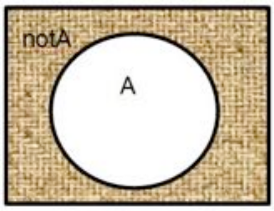
Intended Learning Outcomes At the end of this activity teachers and learners will:

- ✓ Know the vocabulary and discourse associated with probability.
- ✓ Be able to read and process, organise, construct and make links between information and data in contingency tables and Venn diagrams.
- ✓ Appreciate the vital role that language plays in understanding and developing probability concepts and see the power of developing appropriate language to extend existing vocabulary and grammar.
- ✓ Have experienced activities that involve discussion of familiar topics and situations and developing probability models and representations.

Fact Box

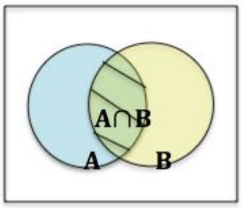
Venn Diagrams

In Venn diagrams the whole set of data being studied (the universe of discourse) is represented by a rectangle. A and B could be attributes (like height or age) describing individuals in a group (as in this chapter) or A and B could be sets of events.



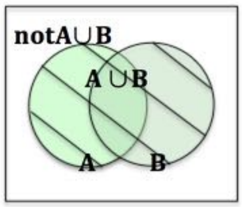
The shaded region, labelled notA, represents the ***complement of the set A***, that is the set of elements that do not belong to A.

If region A represents the set of rational numbers then region notA represents the set of irrational numbers.



The region labelled A ∩ B is the ***intersection of A and B***. The intersection is the set of elements that are both in A and also in B, where sets A and B overlap, shown in green with cross hatch shading.

Example: If A is a set of females, and B a set of people age17 or over, then A ∩ B is a set of females who are age 17 or over.



The region labelled A ∪ B is the ***union of A and B***. The union is the set of elements that are either in A or in B or in both A and B with cross hatch shading.

In this diagram the unshaded region is the complement of A ∪ B, i.e. the elements that are not in either set A or set B.

Example: If A is a set of females, and B a set of people age 17 or over, then notA ∪ B is a set of males who are aged under 17.

Two way contingency table	**A**	**notA**	Totals
B	A and B A∩B	notA and B	
notB	A and notB	notA and notB not(A∪B)	
Totals			

Workshop Activities for Teachers

Activity 1: Where do I fit on the table? Thinking inside the box!!! 90 minutes

Enter the number of teachers in the category for each cell in the table.

1. Each person must decide where they fit on the table – which "box' do they fit into. You can discuss different possibilities. For example "I am a female **and** I am 30 or older" "I am a female **who is** 30 or older" "I am 30 years or older **and** I am a female" "I am **neithe**r male **nor** younger than 30" are all different ways of describing the same event. This discussion is essential in building an understanding and ability to decode contingency tables and Venn diagrams.

2. Gather the data from the group. If it is a large group you may want to do this by a show of hands. Use your language carefully – "Raise your hand if you are a female AND you are under 30" If the group is smaller it can be useful, in terms of identification with the data, for everyone to come up to the board and put their initial in the appropriate square/cell on the table. You can then tally the initials to get the total. If you have sufficient space the participants could form into the four groups. Think about what categories you would use with your learners and how you will collect the data.

	Under 30	**30 or older**	
Male	Number of males who are under 30.	Number of males who are 30 or over.	
Female	Number of females who are under 30.	Number of females who are 30 or over.	

Notes
Statistics is about gathering and interpreting data. Discuss in your group why you think that once the data has been gathered it is very important to have ways of organising this data so that you can work with it. In many sciences and other disciplines the way in which data is organised is in a contingency table. You are going to construct a table using data from the group and together you will change the representation into a Venn diagram. Emphasise the very important role language plays in developing understanding of this way of sorting and representing the data.

Draw the 2 by 2 contingency table shown above. You can change the age boundaries to suit your group. Once you have practised with this table you can construct your own categories. Choose ones that are relevant to the groups you work with.

Checking that you have collected all the data provides the opportunity to discuss the totals that should go in the shaded boxes or cells shown in the table. For example how would the participants check that the data in the table below was correct for their group before the totals were included?

	Under 30	**30 or older**	
Male	2	0	**2**
Female	3	1	**4**
	5	**1**	**6**

Put all the totals in the appropriate boxes in the table you have made for your group. How could you check that everyone was counted? For example, in the table shown, there should be 2 males because 2 + 0 = 2.

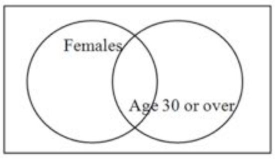
Discussion of mathematical ideas

Questions can now be asked and answered from your contingency table and the totals and the probabilities will be different for different groups. The questions can be about: for example **Numbers of people**: (to ensure you can read the table). How many people are there in the group?(6) How many females are under 30?(3) How many males **are not** under 30?(0)

Fractions of people: to ensure you know what group the person is being selected out of ask, 'is it the whole group or is there a condition?' What fraction **of the group** are male **and** under 30? (2/6) What fraction **of the males** are 30 or older? (0)

Probability There are the foundations of very important probabilistic ideas in these questions : conditional probability; adding the probabilities of mutually exclusive events; the difference between 'and' and 'or' in statements; the idea of complementary events and the use of 'not' in probability. If you choose a person at random from the group what is the probability the person chosen is: a female aged 30 or older?(1/6); is **either** a male under 30 **or** a female of 30 or older? (3/6) What is the chance that a randomly selected female is over 30? (1/4) You can check all these by asking for a show of hands – participants raise their hand if they fit the description.

Activity 2: Changing Representation to a Venn Diagram



On the floor or outside the room make two big circles overlapping like the ones in the diagram, big enough for your group to stand inside. You can use rope or string or just draw lines on the ground. Draw a big rectangle that contains both circles. All the participants should stand inside the rectangle in the space corresponding to their attributes. There are four different spaces in which to stand.

It is important that everyone works out for themselves that some people need to be in the area where the two circles overlap and some need to be outside both the circles. Count how many people are in the same space as you are. This is not as easy as in the table as the place for males is outside the female circle, not inside their own box. Make sure everyone knows where he or she belongs then return to sit down.

When everyone is ready, draw a copy of the circles on the board next to the completed contingency table. Label the circles and fill in the fraction of the group in each space. In the example 1/6 of the group were in the overlap in the diagram as there was one person who was both female and 30 or over. Compare the Venn diagram with the data in the contingency table.

Notes

The outer boundary of a Venn diagram (usually a rectangle) contains all the points representing objects in the sample space (in this case the teachers in your group). The sets of males and females are called complementary because if you are not a female you must be a male and vice versa. You will have worked out that the females stand in their circle and the males stand outside that circle but inside the rectangle.

Ask questions about probability using the Venn diagram. For example: What is the chance that a randomly selected person is: A male who is over 30? Either a female or over 30? Both female and over 30? Try to make up more questions of your own. It is helpful to discuss with other teachers how these activities make connections between language, understanding and representations. You may be able to suggest other questions and other ways to ask them taking into account how learners would express these ideas colloquially or in different mother tongues.

Classroom Activities for Learners

Activity 1: Creating and changing probability representations and collecting data in contingency tables and Venn Diagrams. 1 hour

Initial discussion: "Statistics is about gathering and interpreting data. Once you have gathered the data it is very important to have ways of organising the data so that you can work with it. In many sciences and other disciplines the way in which data is organised is in a contingency table. We are going to construct a table using your data and together we will change the representation into a Venn diagram. It is very important for you to pay attention to the way you use language in your descriptions. This will enable you to understand the language of probability and to be able to read and make sense of contingency tables and Venn diagrams easily" On the board draw an example of a 2 by 2 contingency table. [Choose appropriate age boundaries to suit your group].

	Under 17	**17 or older**	**Totals**
Male	1. Number of males who are under 17	2. Number of males who are 17 or older	
Female	3. Number of females who are under 17	4. Number of females who are 17 or older	
Totals			

What the teacher is doing:	**What the learners are doing:**
Use the techniques in the teachers' activity to: • gather the data • complete the table with the totals for the learners as for the teachers' activity • using the data collected, ask questions that can be answered from the table about learners, fractions and probability.	Deciding which group they belong to: 1, 2, 3 or 4. Answering the teacher's questions. Thinking about different ways the same question can be asked – discussing these in small groups or with a partner.

Teaching Ideas

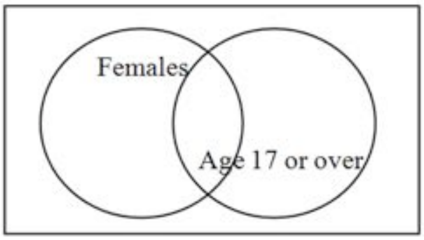
Getting started: You could mark out four rectangles large enough for all the learners to stand in one of them. Label the spaces as in the diagram above. You would need space for this activity. If you are short of space in the classroom you might do this part of the lesson outside.

Draw the diagram on the chalkboard. Encourage learners to discuss which group they fall into and to think of different ways of saying it. Ensure that they know that they can only be in one group. Make sure they understand that **as soon as** they are 17 they will belong to group **2** if they are male but to group **4** if they are female. Take this opportunity to highlight the importance of expressing events accurately. For example, if they are to be in group **4** then two 'events' need to occur: you need to be female and you need to be 17 or older. This is equivalent to: you are not male and are not under 17 (this thinking is useful in the Venn Diagram).

***Encouraging discovery and Statistical / Probabalistic talk**:* Throughout this activity the focus is on learners describing themselves and events in their own language. This will form a solid base on which to develop the more formal language of probability. If this phase is omitted learners will find the probabilistic language of events, unions, intersections etc meaningless to them and only understood it in a rote manner. It empowers learners to realise that they use this sort of language in a range of domains and discussions in their everyday life. For example: You cannot get a driving license unless you are over 18 and you pass the driving test. Ask learners to think of more examples of this sort of language from their usual conversations.

Activity 2: Creating representations of probability. Changing data into a Venn Diagram
Space for learners to form groups, string or chalk to mark circles on the ground. *One hour*

In this activity the learners will form a Venn Diagram based on the same data as they have collected in the contingency table. The diagram on this page uses the data from the table on the opposite page.



On the floor or outside the room make two big circles overlapping like those in the diagram. You can use rope or string or just make a line on the ground. Draw a big rectangle that contains both circles.

Tell the learners that they must all stand somewhere in the rectangle and inside or outside the circles according to their attributes (female or not, age 17 or over, or under 17).

It is important that they work out for themselves that some people need to be in the area where the two circles overlap and some need to be outside both circles. Ask them to count how many people are in the same space as they are.

When everyone has returned to the classroom draw a copy of the circles on the board next to the completed contingency table. Work with the learners to label the circles and to fill in the fraction of the group in each space. For example the group in overlapping part in the diagram are both female **and** 17 or over. Draw attention to the corresponding data in the contingency table.

Teaching Ideas

- *Getting started:* The learners will need to work out which space in the circles is equivalent to the box they were in on the contingency table. It is not always as intuitively obvious, for example ‘being male’ is defined as ‘**not** being female’ and similarly for ‘being under 17’ as ‘**not** being 17 or older’.

- *Encouraging discovery and statistical / probabilistic talk:* Encourage the learners to notice which probabilities from the contingency table are shown on the Venn Diagram and which are not explicitly shown but can be deduced. There is extra or redundant information in the table. However it makes the table much easier to read and understand. Some learners may choose to change their Venn diagrams into tables to make questions clearer.

Discussion of teaching strategy

In these activities it is essential to engage in class discussion about the different wording that can be used, and to make clear the distinction between the different categories being considered, so that learners can use and apply the information with confidence. It is necessary for learners to know, understand, and to be able to use, the standard mathematical terms in the language of instruction (formal English). It is equally important, particularly in data handling, for them to understand how to use their own language to give information accurately. Learners need to understand how alternative forms of wording can be used to convey exactly the same information, for example not being under 17 years old means to be 17 or older and it may be confusing for them that the same information can be given using different forms of language.

Common misconceptions

Even native English speakers may not use the language correctly in everyday conversation or in the same way as the language is used in mathematics. An example of this is the question "would you like tea or coffee?" to which a mathematician would answer "yes' or "no" rather than making a choice between the two because, in mathematics, 'or' is inclusive so logically the question means 'would you like one of these drinks or not?"

The concept of complementary sets is often misunderstood in relation to Venn diagrams. For example sometimes learners make the mistake of drawing two circles, one for males and one for females, rather than one circle to represent one of these sets and the region outside the circle to represent the other.

Key questions to test understanding

	Under 175 cm in height	175 cm in height or taller	Totals
Male	1. Number of males who are under 175 cm in height	2. Number of males who are 175 cm in height or taller	
Female	3. Number of females who are under 175 cm in height	4. Number of females who are 175 cm in height or taller	
Totals			

1. Thinking of your own gender and height which of the 4 boxes in the contingency table are you in? Can you explain why you are in that box and not in any of the other three boxes?
2. Thinking about complementary sets, would you say that the sets of males and females are complementary sets? Why? Would you say that the sets of people under 175 cm in height and of people 175 cm in height or taller are complementary sets? Why?
3. Either using real data from a class in your school or some made up data, make up a contingency table using these attributes. When you have found the totals of males and females what relative frequencies can you calculate and what are they?
4. When you have found the total number of people under 175 cm in height and the total number of people 175 cm or taller what relative frequencies can you calculate and what are they?
5. Suppose you recorded this data from a grade 12 class, would these relative frequencies be the same as if you took a random sample of 100 learners from your whole school? Explain your answer.
6. Draw a Venn diagram to represent the data in your contingency table.

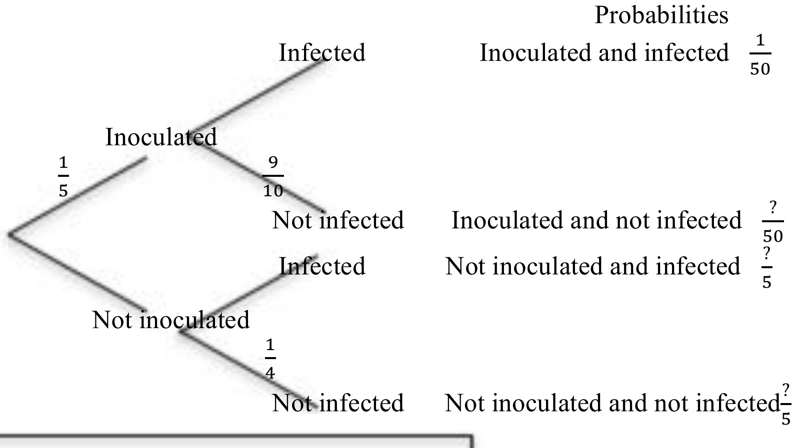
Follow-up Work.

Example for class discussion – The Epidemic
Twenty per cent of the inhabitants of a city have been inoculated against a certain disease. An epidemic hits the city and the chance of infection amongst those inoculated is10% but amongst the rest it is 75%.

1. Fill in the contingency table below and use it to answer the questions.
   a. Put in 20% and 80% for inoculated and not inoculated.
   b. Split the 20% who are inoculated into those who get infected and those who are immune.
   c. Split the 80% who are not inoculated into three quarters who get infected and one quarter who remain immune.
   d. Fill in the totals.

	Inoculated	Not inoculated	
Infected			
Not infected			
Totals			

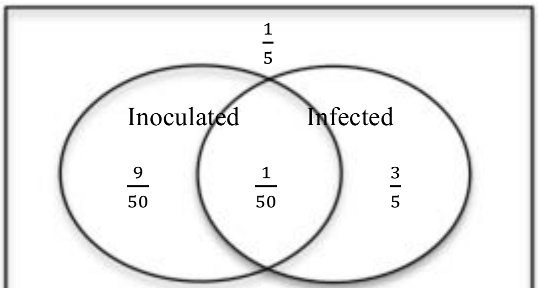
2. What proportion are infected?
3. If a man is chosen at random and found to be infected, what is the chance of his having been inoculated?
4. What is the corresponding chance if he is not infected?
5. Complete the tree diagram below for this question to give the same information as the contingency table in a different way. Fill in the given percentages as fractions to show the probabilities of an individual chosen at random from the population being inoculated or not inoculated and being infected or not infected. (Note: fractions, decimals and percentages are 3 alternative ways of writing the same number.)



6. What proportion of the population are infected?

7. The chance of an infected person having been inoculated is $\frac{1}{50}/\frac{3}{5} = \frac{1}{31}$.

8. What is the chance of a person not infected having been inoculated?



Venn diagram showing the proportions of inoculated or infected people in the city.

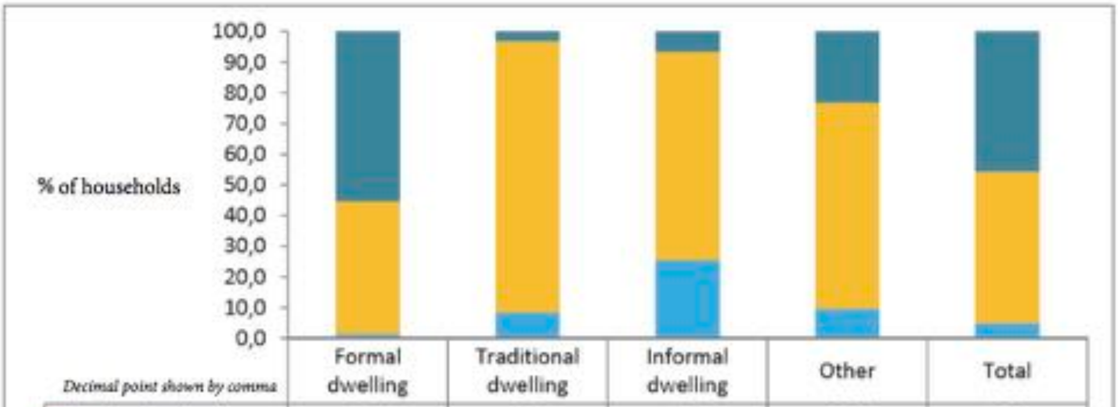
The Venn diagram shows the same information and you can use it to check you answers.

Example for class discussion - Sanitation Study

The UN Millenium Goals include access to adequate sanitation. Sub-Saharan Africa has a very high proportion of the population with inadequate sanitation. The 2016 South Africa Community Survey/Census includes the following diagrams.

What questions can be asked about this data, e.g. access to indoor toilets, type of dwelling etc.? Which representation is most helpful for each question? Why?

Figure 7.12: The distribution of households by location of toilet facilities and type of main dwelling



Decimal point shown by comma	Formal dwelling	Traditional dwelling	Informal dwelling	Other	Total
In the dwelling/house	55,4	3,0	6,6	23,1	45,6
In the yard	43,3	88,5	67,8	67,3	49,5
Outside the yard	1,3	8,5	25,6	9,6	4,9

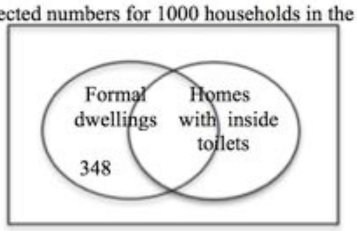
Table 7.15: Distribution of households by location of toilet facilities and type of main dwelling – CS 2016

Toilet location	Formal dwelling		Traditional dwelling		Informal dwelling		Other		Total	
	N	%	N	%	N	%	N	%	N	%
In the dwelling	7 319 207	55,4	32 412	3,0	136 544	6,6	30 818	23,1	**7 518 981**	**45,6**
In the yard	5 724 269	43,3	951 805	88,5	1 400 167	67,8	89 752	67,3	**8 165 993**	**49,5**
Outside the yard	177 250	1,3	91 611	8,5	528 295	25,6	12 866	9,6	**810 022**	**4,9**
Total	**13 220 726**	**100,0**	**1 075 828**	**100,0**	**2 065 007**	**100,0**	**133 436**	**100,0**	**16 494 996**	**100,0**

These statistics can be used to prepare the contingency table below representing 1000 households where the numbers show how many households in the survey fitted each description.
How are these representations related? Explain why the 55.4% of formal dwellings that have indoor toilets in the table above is replaced by 444 households in the table below.

Toilet location	Formal dwelling	Traditional dwelling	Informal dwelling	Other	
Indoor	444	2	8	2	456
In the yard	347	58	85	5	495
Outside the yard	1	5	32	1	49
Totals	**802**	**65**	**125**	**8**	**1000**

Expected numbers for 1000 households in the survey



The same data is represented by four regions in the Venn diagram showing the numbers of households per 1000 that are formal dwellings with and without inside toilets and also homes with inside toilets that are and are not classified as formal dwellings. What does the number 348 represent? Fill in the numbers in the other three regions in the Venn diagram.