AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

## WHO AM I?

How quickly can you find me?

I am a whole number less than 1000.

The product of my digits is 225.

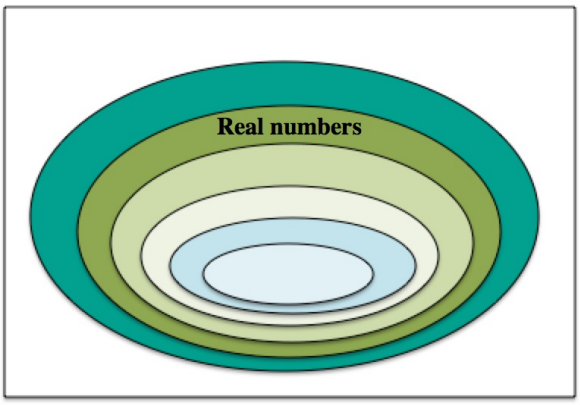
I am not a multiple of 5.

Am I a multiple of 13?

Am I real? Am I rational? Am I irrational?

Now divide me by 11. Is the answer real? Is it rational?

Now multiply me by pi. Is the answer real? Is it rational?




Copy this Venn diagram and label the following sets of numbers:

Integers, Natural numbers, Rational numbers, Whole numbers, Complex numbers, All numbers.

## HELP

Try this first. Cross out the numbers in the squares that do not fit the clues.

Now start the problem above Find the factors of 225 and use the clues to find the number.

Can you find the chosen number from this square using the clues below?

1. The number is odd
2. It is a multiple of three
3. It is smaller than 7 x 4
4. Its tens digit is even
5. It is the greater of the two possibilities

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |

www.nrich.maths.org/content/id/7212/NRICH-poster_OneThirtySix.pdf

## NEXT

Make up a similar number puzzle. Perhaps work with a partner and solve each other's puzzles.

## NOTES FOR TEACHERS

## SOLUTION

**Method 1** 225 = 3 × 3 × 5 × 5 so the digits must be 1, 3, 5 or 9.

To get the product 225 the digits must be 5, 5 and 9 so there are three possibilities: 559, 595 and 955.

The number is not a multiple of 5 so it must be 559.

It is a multiple of 13 as 559 = 13 × 43

**Method 2** Listing all the possible 3 digit numbers, for example: 135,153, 315, 351, 513, 531 etc. We can rule out most of them because their product is not 225. The possibilities are:

111, 113, 115, 119, 135, 139, 159, 133, 155, 199

333, 335, 339, 359, 355, 399

555, 559

999

The only numbers with the product of digits equal to 225 are 559, 595 and 955.

So the answer is 559 as the other two are multiples of 5.

559 is a multiple of 13, it is real, it is rational and it is not irrational.

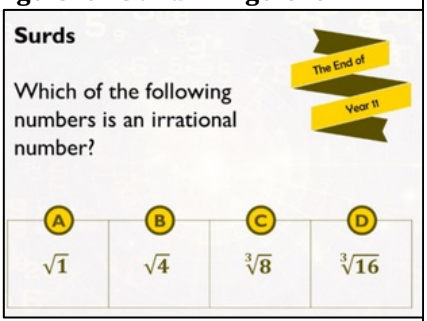
559 ÷ 11 = 50.8181 … recurring in cycles of length 2. This number is real and it is rational.

559π = 1756.15029335669… This number is real and it is irrational. The decimal expansion is infinite and non-repeating.

## Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

**"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**




1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reason for their answer to give them practice in communication.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

**The correct answer is D as the cube root of 16 is $2\sqrt{2}$**

**A.** $\sqrt{1} = \pm 1$

**B.** $\sqrt{4} = \pm 2$

**C.** $\sqrt[3]{8} = 2$

https://diagnosticquestions.com

## Why do this activity?

This activity engages learners in thinking about multiples and factors, and in working systematically to check the different possibilities. It will seem more like a game rather than a routine practice exercise and can become a race to see who finds the number first. It can be used as a lesson starter. It can lead to discussions of the properties of numbers.

## Learning objectives

In doing this activity students will have an opportunity to:

- practise working with multiples and factors;
- develop mathematical thinking and skill of working systematically to check all possible cases;
- develop a deeper understanding of the properties of numbers.

## Generic competences

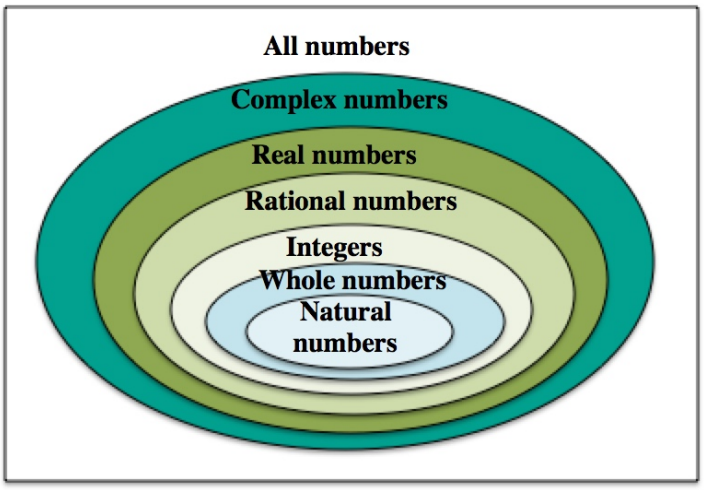
In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **persevere and work systematically** to investigate all possible cases.

## Suggestions for teaching

Start with the diagnostic quiz and a class discussion about types of numbers including rational and irrational numbers.

Write the problem on the board. When several of the learners have found the answer get them to come to the board and explain their reasoning.




Use this example to discuss what the learners know about numbers and use the Venn Diagram to develop their understanding of numbers.

Emphasise that, as the Venn diagram shows, when we learn about numbers we expand our knowledge to a bigger set of numbers that includes all the numbers we already know about.

As small children we started to learn to count (the **natural numbers or counting numbers**).

When we understood that zero is also a number we were working with the **whole numbers**.

When we learnt about negative numbers we were working with **integers**.

When we learned about fractions and decimal fractions we were working with **rational numbers**.

When we learned about numbers like pi and $\sqrt{2}$ we were again expanding our knowledge of numbers to include **irrational numbers** and now working with **real numbers**.

Some quadratic equations do not have real roots but their roots are a different sort of number involving $\sqrt{-1}$ called **complex numbers**.

In higher mathematics and science people work with quaternions, octonions and other numbers.

## Key questions

- Could this be a 2-digit number? Why or why not?
- You know the product of the digits is 225 so what does that tell you about the digits?
- Where would the label go in the Venn diagram for that sort of number?

## Follow up

What are Numbers? https://nrich.maths.org/5805

One of thirty-six https://aiminghigh.aimssec.ac.za/?s=one+of+thirty+six

See the NRICH Poster

www.nrich.maths.org/content/id/7212/NRICH-poster_OneThirtySix.pdf

What do you need? www.nrich.maths.org/5950



Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

| | Lower Primary<br>Approx. Age 5 to 8 | Upper Primary<br>Age 8 to 11 | Lower Secondary<br>Age 11 to 15 | Upper Secondary<br>Age 15+ |
|---|---|---|---|---|
| South Africa | Grades R and 1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| East Africa | Nursery and Primary 1 to 3 | Primary 4 to 6 | Secondary 1 to 3 | Secondary 4 to 6 |
| USA | Kindergarten and G1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| UK | Reception and Years 1 to 3 | Years 4 to 6 | Years 7 to 9 | Years 10 to 13 |