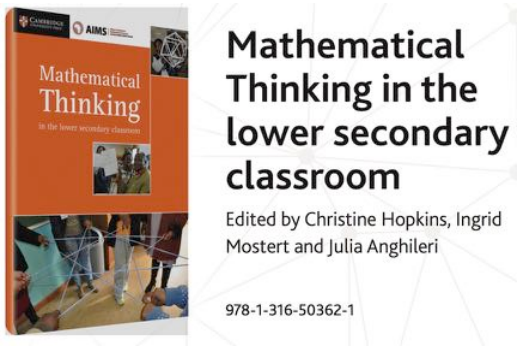
WORKHOP GUIDES FOR TEACHERS TO LEARN TOGETHER
LOWER SECONDARY A3 FUNCTIONS AND INVERSE FUNCTIONS
Guide for your own self-help professional development workshop
and resources for inquiry based lessons.

MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from Amazon or from http://www.cambridge.org/za/education Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to http://www.cup.co.za

For reviews and curriculum map see https://aiminghigh.aimssec.ac.za/mathematical-thinking/

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 **TITLE PAGE**
Teaching strategy.
Curriculum content and learning outcomes.
Summary of mathematical topic (FACT BOX.)

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS
Two pages for you to work through with your colleagues. These are activities to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to stop reading and work on the activity and when to record your work.

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS
Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 TO 10 **CHANGES IN MY CLASSROOM PRACTICE**
Pages on implementing the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates.

Workshop 8 Functions and Inverse functions

Teaching strategy: Visual and getting feedback

Curriculum content: Recognise, describe and represent patterns and relationships. Solve problems using algebraic language and skills.

Prior knowledge needed: Four operations with number, inverse of the four operations.

Intended Learning Outcomes: At the end of this activity teachers and learners will:

- Know how to recognise number patterns and describe them in words
- Understand how the description in words can be written using algebraic notation
- Be able to interpret simple algebraic expressions
- Appreciate patterns in numbers
- Have experienced a variety of activities using a visual learning style.

Fact box

A function is a rule or mapping (often a formula or calculations) that acts on an input to produce an output.
The inverse of a function is a rule that sends the set of outputs back to the inputs.

The inverse of the function 'Add 2' is 'Subtract 2'

The inverse of the function 'Multiply by 4' is 'Divide by 4'

The function n goes to (n multiplied by 2) or n x 2

is usually written as $n \longrightarrow 2n$

The inverse function is written as $n \longrightarrow \frac{n}{2}$

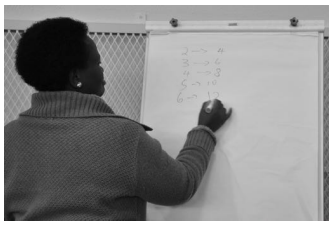
Resources needed for Teacher Workshop: Showboards, large sheet of paper or whiteboard and two differently coloured pens

Workshop Activities for Teachers

Activity 1: Function Game in silence

Board and pens Whole group 45 minutes

Choose someone to be the 'teacher' who will be at the board with the pen. Before beginning it is essential to emphasise that there is no talking and no discussion whilst the game is in progress.
SILENCE IS IMPORTANT! If someone shouts out the pattern the activity is spoilt for everyone else.



Instructions for the 'teacher': Write the following on the board:

$2 \longrightarrow 4$
$3 \longrightarrow 6$
$4 \longrightarrow$

Hold up the pen and invite someone to come and write an answer. You now have to indicate whether the answer is correct or incorrect without speaking. If the number is wrong shake your head and rub it out.
Smile if the answer is correct.

$2 \longrightarrow 4$
$3 \longrightarrow 6$
$4 \longrightarrow 8$
$5 \longrightarrow 10$

After continuing in this way for other different starting numbers a pattern begins to develop. Usually work with consecutive numbers on the left hand side until it is obvious from faces that most people understand. Try this now

Then start to make it a little more difficult by jumping to bigger numbers. At this stage you will become aware that some people are reading the vertical patterns in the table. So if 6 is written – they would write 12 but if 10 is written they would still write 12 because they are seeing the vertical addition of 2 on the right hand side.
But some people will have recognised the rule that is being applied. At this point you need to focus on those people who have not yet spotted the pattern and build the pattern again.
Use numbers that make the rule obvious. So write:

$20 \longrightarrow 40$
$30 \longrightarrow 60$
$40 \longrightarrow 80$
$50 \longrightarrow 100$

Ask for the rule in words (speaking is allowed now). Likely responses are 'double the number' or 'multiply by 2'. Write number $\longrightarrow$ number x2.
Hold out the pen again and invite someone to write something on the board taking n for the first number so $n \longrightarrow n \times 2$ or $n \longrightarrow 2n$.

Now take turns to choose a function. Write it down without letting anyone else see. Take it in turns to give the lesson as above but using your own secret function. Continue giving examples until everyone in the group can guess the rule. Then ask for the rule to be written as $n \longrightarrow ?$

Activity 2: Extending the function game

Large board, pens Pairs, whole group 45 minutes

Functions that can be written in more than one way.

The function "double the number and add 2" can be written as

$$n \longrightarrow 2n + 2$$

When you play the function game some learners may spot the pattern as 'Add one to the number and double it'

$$n \longrightarrow (n+1) \times 2$$

or $$n \longrightarrow 2(n+1)$$

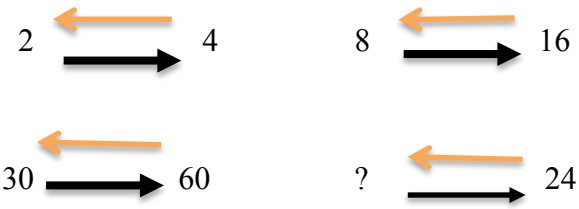
This describes exactly the same function because

$$2n + 2 = 2(n+1).$$

Everyone in the group should choose a secret function which can be written in more than one way and present it to the group in silence as described in Activity 1: Function Game. Continue until there are lots of examples on the board and everyone is confident of the function before allowing anyone to speak. Then ask for the function to be described first in words and then as $n \longrightarrow$? The group should be able to find two different ways of describing each function.

Inverse functions

When you have finished a function game. Take a different coloured pen and draw arrows back from the final number to the starting number. These arrows show the Inverse function.



The function is "double the number", which can be written $n \longrightarrow 2n$

The inverse function is "divide the number by 2", which can be written

$$n \longrightarrow n/2$$

Everyone in the group takes the lead in being the teacher and introducing a function and then, by drawing arrows backwards, the inverse function.

Notes

- Experiment with slightly different ways of presenting the game. What happens if you choose random starting numbers instead of a sequence of numbers? Which numbers seem to be helpful in guessing? Try scattering the numbers around the board rather than lining them up in a column. Try including fractions or decimals.
- Lots of examples make it easier to guess the rule – around 20 may be needed.
- You may like to draw a smiley face on the board for a correct answer or hold up a showboard with a smiley face.

Classroom Activity for Learners

Activity 1: Function Game

Large board | Whole class | 50 minutes

What the teacher is doing	What the learners are doing
Explain to the class that they must be absolutely silent. Choose a function and writes the first few examples on the board. Continue the activity as described in Teacher Activity 1. Add lots of numbers. Try large starting numbers and possibly fractions. Continue the game in silence until you think almost everyone has guessed the rule. Tell the learners they are **now allowed to speak** and ask for the rule in words. Ask if anyone can write the rule more shortly using n for the starting number.	Learners watch the board. It should be very quiet and the learners should not put up their hands. 1 → 5 2 → 6 3 → 7 4 → 8 8 → ? The teacher walks around the class and offers the pen to a learner. The learner enters their guess for the number and hopes to get a smile. After some time learners are allowed to speak. They may say: You add on 4 each time *or* number → number plus 4 *or* n → n + 4

Teaching ideas

- The initial objective is that learners will:
  - Recognise number patterns and represent them by a function.
  - Use letters to represent numbers.
- Play the game using all the ideas in Teacher Activity 1. Learners will need lots of practice with single step functions - that is additions, multiplications, subtractions and division, before going on to the two-step functions.
- The box shows a sample two-step function. Do you have enough information in the box to decide what the function is? Learners will often need around 20 examples on the board before everyone has guessed the function.

1 → 1
2 → 3
3 → 5
8 → 15

20 → 39
1000 → 1999

What is the rule?

You double the number and take away one
number → number x 2 then take away 1
n → 2n -1

Activity 2: Extending the function game

Large board and pens | Whole class | 50 minutes

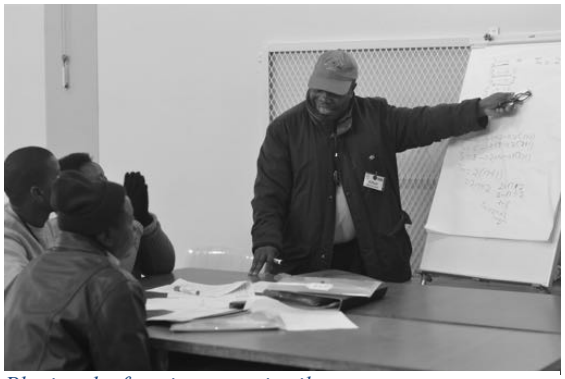
What the teacher is doing	What the learners are doing
Functions that can be written in more than one way	
Choose a **function that can be written in more than one way**. Keep adding examples until you think everyone has guessed the rule.	Learners guess the function. Here are some examples: 5 → 18 6 → 21 7 → 24 8 → 27 10 → 33 20 → 63 100 → 303
Explain that both expressions are correct. (n+1) x 3 is usually written as 3(n+1) and 3(n+1) means exactly the same as 3n+3.	Some learners may guess the rule as 'three times the number plus 3' n → 3n+ 3 **or** 'add one to the number and multiply by 3' n → (n+1) x3
Inverse functions	
Play the function game with a one-step function e.g. 'multiply by 3'. Leave all the examples on the board.	Learners guess the function. 'number is multiplied by 3' n → 3n 5 → 15 ← ? → 30
Tell the class they must be silent again and concentrate. This time write the **output** number on the board and hand the pen to a learner to fill in the **input** number. After a while ask each time for the inverse function so that learners add this word to their mathematical vocabulary. Repeat for other functions	Learners know the function is multiply by 3 but now it is not the output number they have to guess but the input number. Learners describe the inverse function, first in words Divide by 3. Then as n → n ÷ 3 Or n → $\frac{n}{3}$ or ⅓n

Teaching Ideas

- Don't tell the learners that there is more than one way to describe the rule just ask if anyone has a different way of describing the rule. If you get two different formulae, ask the class which one they think is correct. Get them to explain to you that both are correct! It greatly boosts the confidence of a learner if they are doing the explaining rather than the teacher.
- Drawing another arrow in a different colour back from the output to a question mark for the input emphasises that the inverse function takes you back to where you started.

Changes in my classroom

Implementing the teaching strategy



Playing the function game in silence

Visual learning

What is the point of the silence? This is a learning device. All learners come to an understanding of the rule at different stages. Calling out the rule spoils that moment at which the child realises that they understand. The learners are **starting from a problem not a technique**. The silence helps you to read faces and it is very clear when they understand so this enables you to **find out which learners understand**. Because there is no auditory input the learners must concentrate on the **visual**.

Kinaesthetic learning

The silence makes it dramatic as an individual learner comes to the board and writes. As you walk around the class and offer the pen you will learn to judge when learners are ready to write on the board. You can stage-manage the situation so that confident learners are given the pen for a complicated number and less confident learners for simpler ones.

Adapting the activity for a large class

If your class is large you can involve more learners by using showboards. Explain at the beginning that they must be silent and that when you point to the whiteboard they must write on their showboards the number they think goes at the end of the arrow. Point to someone with the correct answer to come and write it on the board or get them to pass their board forward if movement around the classroom is difficult. You can show the showboard to the class and write the number on the large board yourself. You will not need to rub out any answers as you can always choose someone with the correct answer. You will also have a very good idea of how many learners have guessed the pattern.

Key Questions to develop understanding

This is a silent activity so the teacher uses few questions! Instead of spoken questions the teacher writes on the board and offers the pen. The crucial questions at the end of the activity are:

- What is the rule?
- Can you write it differently?
- Can you write the rule more shortly by using n to stand for the number?

Follow up Activities

One and two stage inverse functions

Once the learners are confident that

addition and subtraction are inverse operations

multiplication and division are inverse operations

you can work on the inverse of two stage functions. If the learners are good at guessing functions then they should be able to guess that

$1 \longrightarrow 3$
$2 \longrightarrow 5$
$5 \longrightarrow 11$
$6 \longrightarrow 13$ is $n \longrightarrow 2n + 1$.

Now try that backwards

$3 \longrightarrow 1$
$5 \longrightarrow 2$
$11 \longrightarrow 5$
$13 \longrightarrow 6$

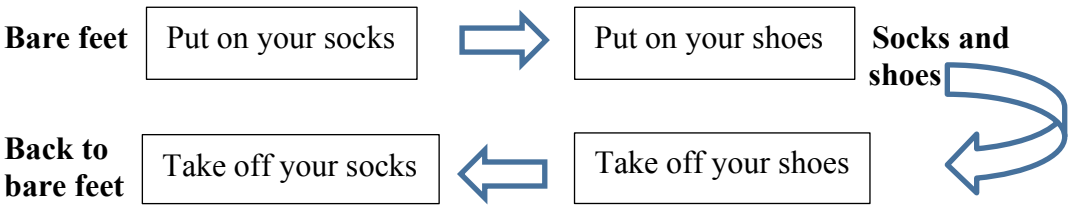
Dividing by 2 seems an obvious idea – but that gives fractions.

Taking away one first and then dividing by two is what you need $n \longrightarrow (n - 1)/2$

We all know that the inverse of 'putting on your socks and your shoes' is NOT

'taking off your socks and then taking off your shoes'.

Flow diagrams work well to demonstrate this



Get the learners to think of more examples. For example:

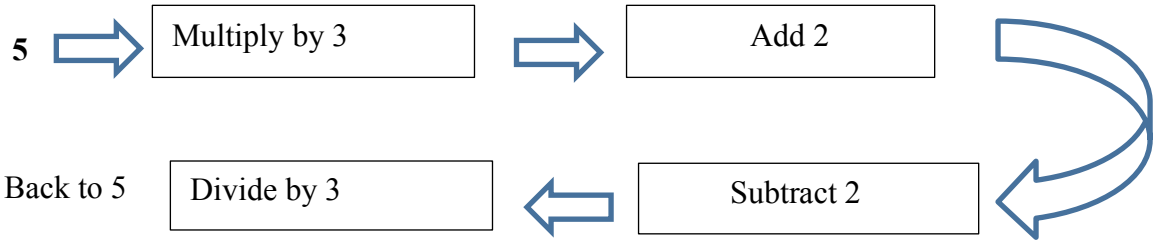
put on your school uniform - go to school – come home from school – take off your uniform.

go to the moon - land on the moon - take off from the moon - come back to earth

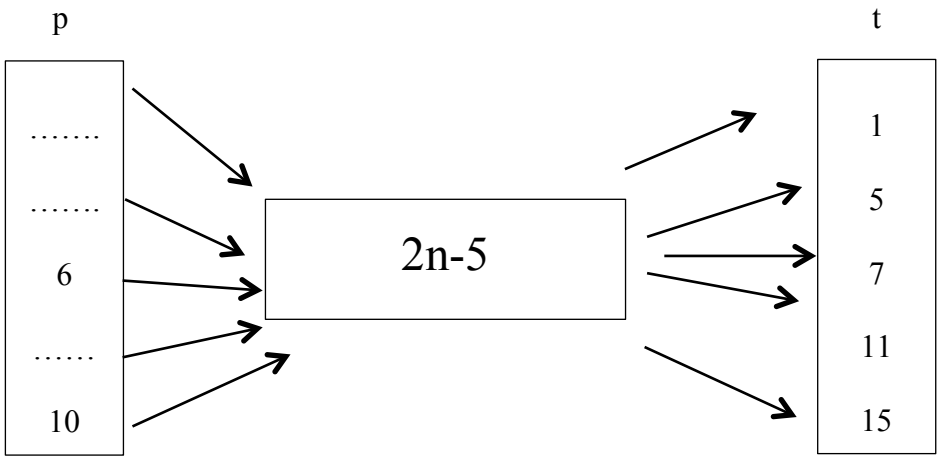
As long as both actions have an inverse this will work:

stand up - leave the room - come back into the room - sit down

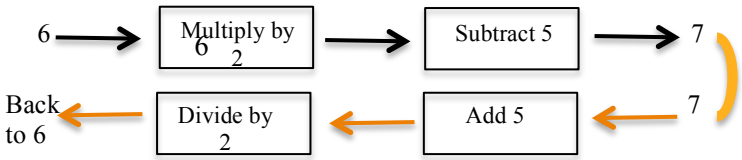
It is like running a film backwards: you undo the last thing you did so



Use the idea of inverse operations to find the missing numbers



To solve this problem first try 'running the film' forward using the number 6 which the function 2n – 5 sends to 7.



Then run the film backwards, the inverse of subtract 5 is add 5 and the inverse of multiply by 2 is divide by 2. So the inverse function is $n \rightarrow (n + 5) \div 2$ or $\frac{1}{2}(n + 5)$.

For follow up work see:
Mind Reader https://aiminghigh.aimssec.ac.za/grade-9-mind-reader/
Building Functions https://aiminghigh.aimssec.ac.za/grades-9-to-12-building-functions/
Undoing https://aiminghigh.aimssec.ac.za/grades-9-to-12-undoing/