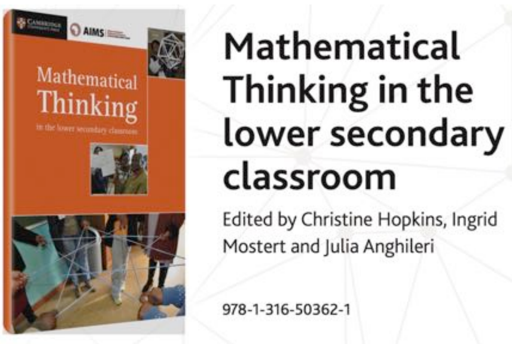
MANAGE YOUR OWN PROFESSIONAL DEVELOPMENT WORKSHOP

These guides are designed to support teachers in developing a deep understanding of the mathematics they teach and in developing more effective ways of teaching.

You can use these guides on your own or as one of a group of teachers who meet together to talk about your mathematics lessons as part of your professional development. Maybe one of you will take the lead in organizing time, date and venue but once you are doing the activities together you will all participate on equal terms in the discussion and reflection.



These Lower Secondary Workshop Guides are chapters in the AIMSSEC Mathematical Thinking Book.

Buy the book online from Amazon or from http://www.cambridge.org/za/education Search for AIMSSEC or for ISBN 9781316503621. To order the book in South Africa go directly to http://www.cup.co.za

For reviews and curriculum map see https://aiminghigh.aimssec.ac.za/mathematical-thinking/

EACH WORKSHOP GUIDE HAS A SIMILAR FORMAT:

PAGE 1 **TITLE PAGE**
Teaching strategy.
Curriculum content and learning outcomes.
Summary of mathematical topic (FACT BOX.)

PAGES 2 & 3 WORKSHOP ACTIVITIES FOR TEACHERS
Two pages for you to work through with your colleagues. These are activities to be shared and discussed. For each activity there is a list of resources needed, how to organise the activity (e.g. individual, pairs, whole class), how long the activity will take, when to stop reading and work on the activity and when to record your work.

PAGES 4 & 5 CLASSROOM ACTIVITIES FOR LEARNERS
Two pages to help you plan your lesson. You are advised how long to allow for the activity, the resources you might need and the key questions to ask.

PAGES 6 TO 10 **CHANGES IN MY CLASSROOM PRACTICE**
Pages on implementing the teaching strategies with additional resources and activities for use during or after the workshop such as worksheets and templates.

Sequences and patterns

Teaching strategy: Visual and practical

Curriculum content: Recognise, describe and represent patterns and relationships.

Prior knowledge: Understand the use of a letter to stand for a variable.

Intended Learning Outcomes: At the end of this activity teachers and learners will

- Know what an algebraic expression means
- Understand the difference between the variable and the constant in an expression
- Be able to construct patterns to represent algebraic expressions
- Be able to find a formula to represent a growing pattern
- Appreciate the value of giving meaning to an algebraic expression
- Have experienced a practical activity to support their understanding

Fact box

A **variable** can take different values in an algebraic expression. Letters are used to represent variables.

A **coefficient** is a number by which variables are multiplied. A coefficient acts on a variable.

A **constant** stays the same in an algebraic expression.

Example: $4n + 1$	n is a variable.	It takes different values.
	4 is a coefficient.	It tells you how many of n you have got.
	1 is a constant.	It does not change.

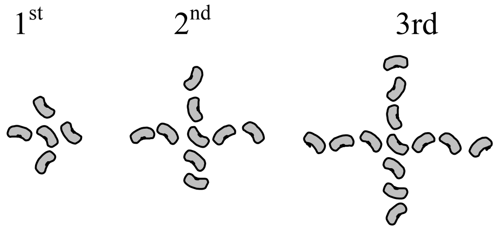
Resources for this workshop: Lots of small objects, You will need two different sorts of small objects e.g. beans and buttons or red and blue counters

Workshop Activities for Teachers

Activity 1: Making sequences with bean counters

Beans or similar counters e.g. buttons, stones, seeds | Small groups or pairs | 1 hour

1. One teacher in the group should arrange the counters into this sequence of patterns.

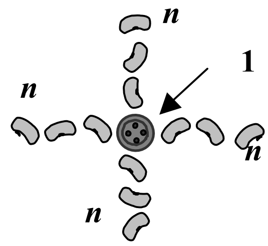


How many counters are there in each pattern?
How is the pattern growing? Focus on the structure of the growing pattern.
Another teacher should give precise instructions describing how to make the 8th pattern.
T= TRY this now
2. Can you predict the number of counters in the 4th pattern? 5th pattern? 10th pattern? 100th pattern? T
3. Find an expression for the number of counters in the n^{th} pattern in the sequence. T
4. Work in pairs to create a sequence of patterns in which the n^{th} pattern has $2n + 3$ counters. Discuss different series of patterns produced. T
5. Go on to create sequence of patterns for more complex algebraic expressions. T

Notes
1. The pattern grows by adding four more counters - one on each 'arm' are added to the previous pattern with one counter in the middle. To make the 8th pattern put 8 counters on each of the 4 arms and one in the middle

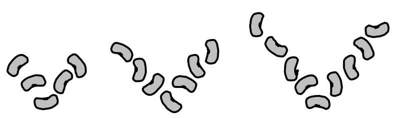
2. Numbers of counters: 4th pattern: 17; 5th pattern: 21; 10th pattern: 41; 100th pattern: 401

3. $4n + 1$ Make sure you understand the relationship between the structure of the pattern and the algebraic expression. You could replace each middle counter in the patterns with another one of a different type or colour, to emphasise its position as the '+ 1'.



4. Try to find several different possible arrangements.

5. Focus on the structure of the patterns. Do some arrangements bring this out more clearly than others? For example, it is easier to see the growing 'arms' in this pattern:



than in this one:

Activity 2: Bean counters and algebra

Beans or similar counters e.g. buttons, stones, seeds | Pairs, whole group | 45 minutes

1. Arrange counters into this sequence of patterns:

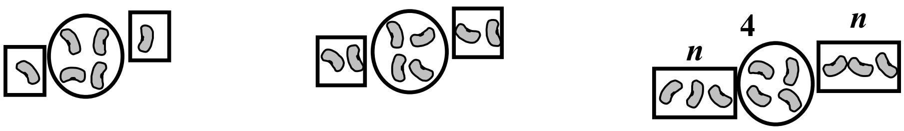
1st 2nd 3rd

Find an expression for the number of counters in the n^{th} member of the sequence. Can you find more than one way of writing this expression? T

2. Discuss how different ways of looking at the structure of the patterns can lead to different forms of algebraic expression. T

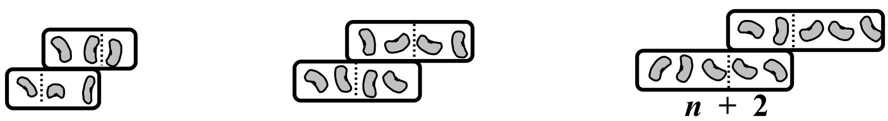
$2n + 4$

a.



b.

$2(2 + n)$

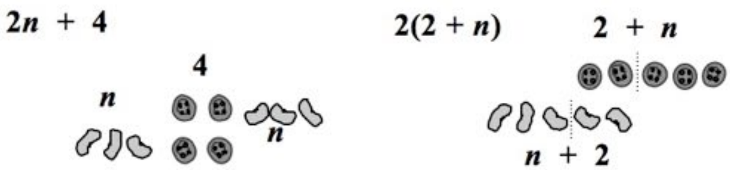


3. Make a sequence of counters to show 4n+4. Can you see from your pattern that this is also 4(n+1)? T

Notes

2a. $2n + 4$ will probably be the first suggestion.
A constant 4 in the circle and two arms with n counters which grow longer and longer.
2b. Looking at the pattern as two lines gives the formula $2(2 + n)$.

3. You can draw loops around the counters to show the two ways of looking at the arrangement, as above. Alternatively, use different counters to replace some parts of the pattern, one counter at a time. Be careful to show that you are not changing the pattern, just the items that you are using as counters.



When n=3 there are two ways of using different counters. The first diagram emphasises 4 counters in the middle staying the same for the formula 2n + 4.
The second diagram emphasises two rows each with the same number (n + 2) of counters.

Classroom Activity for Learners

Activity 1: Counting beans

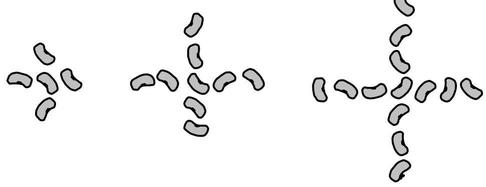
Two sorts of counter e.g. beans and buttons. About 50 counters for each group

Pairs or small groups

50 minutes

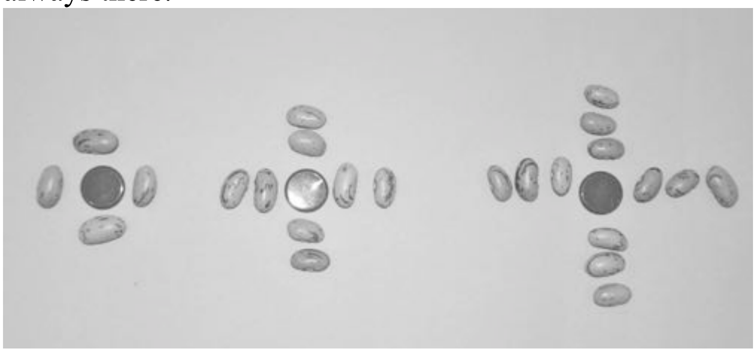
1. Draw the first series of patterns on the board

1st 2nd 3rd



Ask learners to copy the series with their own counters and to write down the number of counters in each pattern.

2. Ask how the pattern is growing. Focus on the structure of the growing pattern. Ask a learner to give precise instructions for drawing the 8th pattern.
3. Ask learners to predict the number of counters in the 4th pattern and 5th pattern.
4. Ask one or two learners to explain their reasoning.
5. Ask learners to predict the number of counters in the 10th and the 100th patterns.
6. Ask one or two learners to explain their reasoning.
7. Demonstrate the structure of the growing patterns by replacing the middle counters with an alternative, different one. Talk about the way that the 'arms' grow, but the counter in the middle is always there.



Ask learners to replace the middle counters in their own patterns.
8. Introduce the expression $4n + 1$ for the number of counters in the nth pattern in the sequence.
9. Show learners a series of patterns in which the nth pattern has $2n + 3$ counters. Ask them to copy the patterns, then ask what the algebraic expression could be. Encourage them to see that $2n + 3$ and $3 + 2n$ are both correct. Again, emphasise the relationship between the structure of the patterns and the algebraic equation.
10. Ask learners to create a series of patterns for $3n + 4$. Compare different layouts of the patterns. Establish that they all have the same underlying structure.
11. Work on further series of patterns.

Teaching Ideas

1&2 Notice how after looking at the 1st, 2nd and 3rd patterns there is a jump to ask about the 8th pattern. This jump is really important because you want the learners to find the **formula** for the sequence not just the next term. Learners need to notice that in the 2nd pattern the arms have 2 beans, in the 3rd pattern the arms have 3 beans so in the 8th pattern the arms will have 8 beans.
3&4 The learners should have two ways of working these out: by adding 4 counters to the pattern each time and by knowing that in the 5th pattern each arm will have 5 beans so 4x5 + one in the middle.
5&6 Adding 4 takes too long for the 100th but 4x100 + 1 will give you the answer

Activity 2: Matching Patterns to Expressions

Worksheet on page 8, scissors | Pairs or small groups | 30 minutes

Photocopy the worksheet of patterns and expressions on page 8 for the learners. The learners should cut out these expressions for the nth term and match them to the correct sequence on the worksheet.

$3n + 4$ $4n - 1$ n^2 $4n + 2$ $2n^2 - 1$ $2n + 1$

As the groups match the expressions to the patterns, ask them WHY they have chosen the expression for that pattern. Can they see the connection between the formula and the number of 'arms' to the pattern? Is there a connection with the number of shapes at the centre of the pattern?

Teaching ideas

- Ask learners who finish quickly to choose one of the patterns and:
  - Draw the next pattern in the sequence.
  - Work out how many counters in the 10th and the 100th pattern.
- You will find more questions to ask on page 7.

Changes in my classroom

Implementing the teaching strategy

Making algebra practical and visual

You can use small objects and diagrams to model a mathematical sequence. A sequence of patterns can offer a concrete, physical representation of the abstract algebraic expression. Visualising the patterns gives the learner a 'model to think with' which will support their understanding of what is happening when they manipulate algebraic expressions.

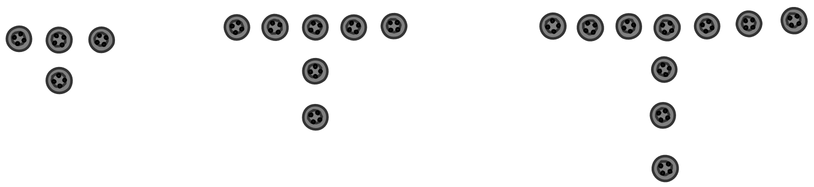
Many learners find it difficult to put any meaning into an abstract algebraic expression. They may learn some techniques for manipulating expressions and equations, but without a basic underlying understanding of what the variables and the numbers represent this may be quite meaningless. This may lead to errors as learners forget the 'rules' for manipulating algebra.

There is no one 'correct' representation of any particular expression. Different learners will come up with different patterns for the same expression. They should be encouraged to look at several patterns for each expression, and to discuss what is different and what is the same.

Changing the middle counter helps learners to visualise the pattern as 4 x arms plus 1. If you now ask for several large easy numbers 20^{th} , 30^{th} , 1000^{th} then some learners may be able to say that for any number (n^{th}) the number of beans is 4 times the number plus 1 or 4n+1.

Differentiation

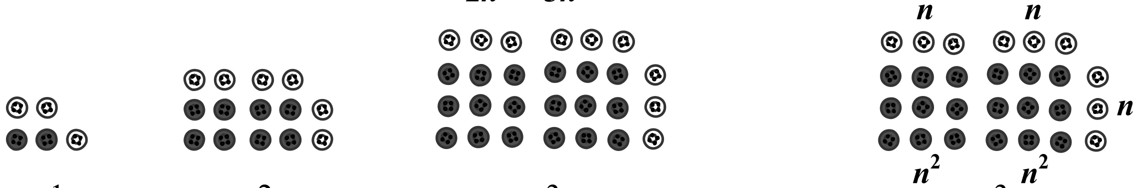
Using practical apparatus can make it easier to differentiate the task. All learners can be asked to make the first few patterns for a sequence. Learners can choose to make simple patterns based on letters of the alphabet such as T or X or Y.



Learners who want a challenge could create patterns for expressions such as $2n^2 + 3n$.
It may be easier to start by building the third or fourth pattern in the series in order to see how they are structured, and then working backwards to the earlier patterns.
In the n=2 pattern, can you spot the two 2x2 squares?

showing structure

$2n^2 + 3n$



n=1 n=2 n=3 n=3

Key Questions to develop understanding

You can ask the learners the following questions in relation to any sequence of patterns for which there is an algebraic expression for the nth term.

- Draw the next pattern in the sequence.
- Explain how to draw the next pattern in the sequence.
- How many extra objects would you use to change this pattern into the next pattern in the sequence?
- Tell me how many objects there are in each of the first four patterns in the sequence. Do those numbers follow a pattern?
- Could you tell me how many objects there are in the next pattern in the sequence without making or drawing the pattern? How did you work that out?
- How would you find out how many objects there are in the 10th pattern in the sequence?
- How would you find out how many objects there are in 100th pattern in the sequence?
- Can you find an algebraic expression (formula) for the number of objects in each pattern in the sequence?

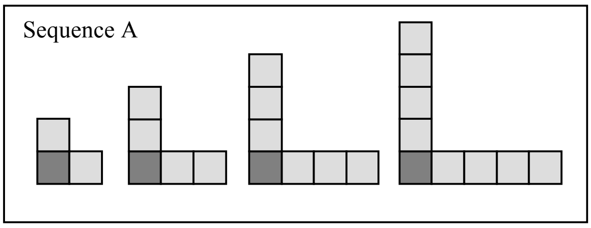
Helping learners to remember

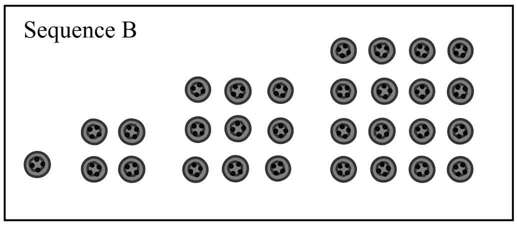
Posters
Groups of learners could create posters showing how the structure of one sequence of patterns relates to the expression for the n^{th} pattern in the sequence. Having some posters on the wall and talking about them occasionally will help to fix the ideas. Give the group squared paper and the triangle dotty paper so they can draw several sequences. Each poster should have a question e.g. How many triangles in the 10^{th} shape?

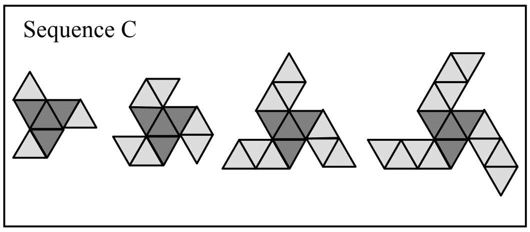
Mental Mathematics
What is the value?
Write a formula in the middle of the board: 4 + 3n.
Give the learners a number for n and ask them to work out the value of the expression. If you have showboards ask everyone to show you their answer and you will know immediately if they understand. If you don't have showboards the learners can write down their answers and mark each other's after you have given several values for n.

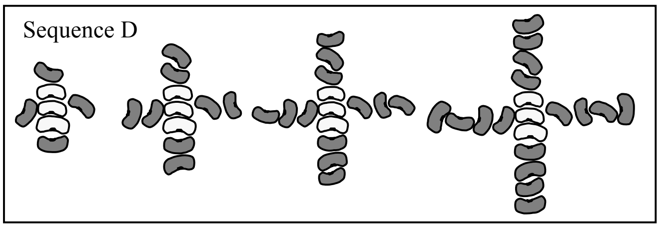
Quiet counting
Use the expression 4 + 3n and draw the first few patterns using counters. Ask the class to guess whether any of the patterns in the sequence will have exactly 100 counters. Show hands for yes, show hands for no. Encourage the learners just to make a guess – they won't be sure at this stage Now very quietly lead the class in counting 7, 10, 13, 16, 19, 23, ………….and continue to see if you hit 100.
A few days later you can repeat with different numbers. These regular few minutes of mental maths at the beginning of the lesson can really help learners to remember ideas

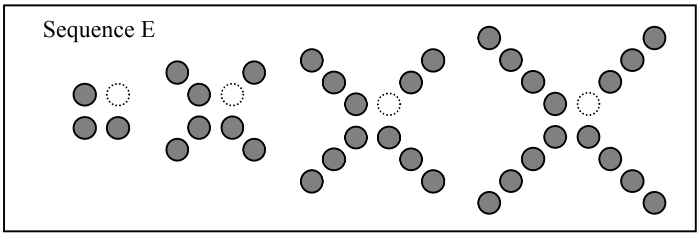
Match each sequence to the expression for the *n*th pattern in the sequence.

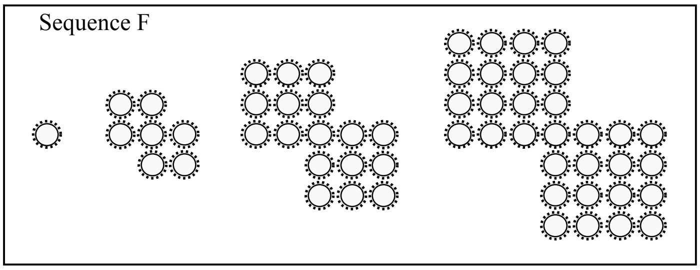












Expressions for the nth pattern. Cut out and match to the correct sequence

3n + 4	**4n + 2**	**n^2**
4n	**$2n^2 - 1$**	**2n + 1**