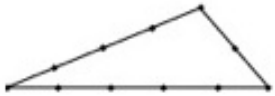
MAKING TRIANGLES



Use 11 sticks of equal length to make this triangle with edge lengths 2, 4 and 5.

You might like to record this as (2, 4, 5).

How many other triangles can you make with 11 sticks?

Investigate all the triangles that you can make with different numbers of sticks.

Record your results in this table.

Number of sticks	1	2	3	4	5	6	7	8	9	10	11
Triangles											

Is it possible to make a triangle with edge lengths 2, 3 and 6?

What about 2, 3 and 4?

When can you make a triangle and when is it impossible? What patterns can you find?

To investigate this question you could use toothpicks, or paper sticks or a piece of string with equally spaced knots.

To make 4 paper sticks each 10.5 cm long, cut a piece of A4 paper in four, roll the paper with the string inside so that you can join the sticks by tying the string.

HELP

You can make a triangle with 3 sticks but not with 2 sticks or 4 sticks. Try it and explain why.

Make chains of sticks and make triangles with them. Record the triangles you find by 3 numbers for the lengths of the edges, for example the triangle shown is (2, 4, 5).
To be sure that you have found all possible cases you need to make a systematic plan.

NEXT

Wholesome Rectangles
https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/
is about exploring the possibilities for rectangles with whole number dimensions and finding their perimeters and areas.

Resources: Paper sticks

NOTES FOR TEACHERS

SOLUTION

With 11 sticks you can make 4 different triangles as shown in the last column of the table below.

Number of sticks	1	2	3	4	5	6	7	8	9	10	11
Triangles	None	None	(1,1,1)	None	(1,2,2)	(2,2,2)	(1,3,3) (2,2,3)	(2,3,3)	(1,4,4) (2,3,4) (3,3,3)	(2,4,4) (3,3,4)	(1,5,5) (2,4,5) (3,3,5) (3,4,4)

Even though 2 + 3 + 6 = 11 it is not possible to make a (2, 3, 6) triangle because 2 + 3 < 6.

For 3 lengths to give a triangle the rule is that the sum of the lengths of the any two edges must be greater than the length of the third edge.

For lengths a, b and c to make a triangle: a + b > c, b + c > a and c + a > b.

This is called **the triangle inequality**. Check that it works for your triangles.

A (2, 3, 4) triangle with a perimeter of 9 units can be made.

DIAGNOSTIC ASSESSMENT

Use at the end of the lesson. This should take only 5–10 min.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Which of the following sets of 3 lengths cannot make the edges of a triangle?

- **A. 3, 4, 6**
- **B. 3, 4, 8**
- **C. 5, 12, 13**
- **D. 4, 5, 6**

1.Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2.It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4.Ask the class to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: B. Join the edges of lengths 3 and 4 units.
It's impossible to make the third edge of the triangle longer than 7 units.

Why do this activity?

This activity reinforces the understanding of perimeter and it is accessible to all abilities. Weaker learners can be successful in making triangles and counting the sticks. Some of the ablest learners may discover the triangle inequality through Activity 1.

Rather than just hearing about it from somebody else, discovering the triangle inequality should be a satisfying experience and boost learners' confidence. Modelling

the mathematical concept is powerful because it is intuitively obvious from using the sticks that certain triangles cannot be made but it is quite difficult to think about in the abstract.

The **general teaching strategy** here is **modelling mathematical concepts using manipulatives** so that learners can easily change the shapes and explore the possible configurations. Handling the manipulatives helps learners to visualise the mathematical shapes and their properties. The objective is for learners to progress to being able to visualise the shapes without using the concrete materials and to be able to work from the numbers and the properties of the shapes where different numbers are involved.

Learning objectives

In doing this activity students will have an opportunity to gain a deeper understanding of the concept of perimeter.

Generic competences

In doing this activity students will have an opportunity to develop problem solving, mathematical thinking and systematic working skills.

Suggestions for teaching

The stringy stick loops are very suitable for holding up in a lesson for the whole class to see. To aid a class discussion, teachers can ask learners to come to the front to make triangles with the loops, then ask other learners to draw the shapes on the chalkboard. You might use longer sticks (21 cm) with a big class and 10.5 cm sticks for group work. Toothpicks, or used matchsticks, may be more convenient for individual or group work.

You can write the question on the chalkboard or print out a worksheet Copy part of page 1) and give a copy of the HELP section to learners if they are struggling to get started.

You can do this as a whole class activity or organise the class to work in pairs or groups.

1. First ask 2 learners to come to the front and to make the (2,4,5) triangle to show the class. Then ask them to make a different triangle. Then split the class into groups, give each group an 11 stick stringy loop or toothpicks and ask them to make and record as many different triangles as possible.
2. Ask learners to explain why it is impossible to make a triangle with the sticks with perimeter 4 units.
3. Explain that this activity is about exploring the possibilities for triangles with whole number dimensions. With other lengths there are infinitely many possible triangles so here we just stick to whole numbers. In order to find all the triangles with a given perimeter learners need to **work systematically** and this is an important mathematical skill.
4. Ensure that all learners DO the activity, TALK about it and RECORD their working, that is that they engage in all three of the '**Do, Talk, Record**' actions in each lesson.
5. Check that learners are recording all their findings. It is important **to record results**

clearly and systematically, another important mathematical skill. The results can be recorded using diagrams and numbers to represent the edge lengths or using a table. Depending on your class you might let learners find their own ways to record results or you might suggest using a table.

6. Finish the lesson with a class discussion of all the results that the class has found and record them systematically. Have a class discussion about the advantages of different methods of recording if you let them choose their own methods.
7. Discuss the rule for when it is possible to make a triangle and why it is impossible for some sets of three lengths.
8. You may like to get learners to copy notes into their workbooks like this 'Fact box'.

Fact box

Perimeter

The perimeter of a shape is the distance all the way around the edges, the total length of the boundary. For example, if a farmer puts a fence all the way around a field, of any shape then the length of fencing needed is the perimeter. For a polygon the perimeter is the sum of the lengths of the edges. Perimeter is measured in centimetres, metres or kilometres.

Area

The area is the amount of space inside the boundary of a 2-dimensional object such as a triangle, a square or a rectangle or any shape such as this squiggle.

Area is measured in square centimetres, square metres or square kilometres. There are different ways to work out the areas of different shapes including integral calculus where we imagine the area made up of infinitely many infinitely thin rectangular strips.

Triangle Inequality

For 3 lengths to make the edges of a triangle the rule is:
The sum of the lengths of any two edges is greater than the length of the third edge.

Lengths *a*, *b* and *c* make a triangle if and only if *a* + *b* > *c*, *b* + *c* > *a* and *c* + *a* > *b*.

Key questions

- What can you tell me about the shape you have made?
- How did you decide how to record your data?
- How do you know if you have found all the possibilities?
- Can you find any patterns?

- Can you carry on and find more results using bigger loops or using bigger numbers without using a loop?
- How would you describe the area and perimeter of your shape?
- What do you notice? What is the same? What is different?
- List all the combinations of 3 whole numbers that add up to 12 (for example 1 + 4 + 7). Which of these number triples can make the edge lengths for a triangle? Give reasons.

Follow up

Wholesome Rectangles
https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/
is about exploring the possibilities for rectangles with whole number dimensions and finding their perimeters and areas.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13