

MAGIC NUMBERS

Work out these subtractions.
Then divide each answer by 8.
What patterns do you see?

$9 - 1 =$
 $98 - 2 =$
 $987 - 3 =$
 $9876 - 4 =$
 $98765 - 5 =$
 $987654 - 6 =$
 $9876543 - 7 =$
 $98765432 - 8 =$
 $987654321 - 9 =$

Suppose you share 9872 gold coins between 8 people.
You split them into 4 piles, as shown in the picture,
chosen so that each pile is a multiple of 8.
Then you share each of the piles between the 8 people.
You give each person $1000 + 200 + 30 + 4$ coins.



Use the coin sharing story to explain the division sum:

$$9872 \div 8 = 1000 + 200 + 30 + 4 = 1234$$

The brackets in these calculations tell you that you must do the calculation inside the brackets **first**.

Using the subtraction calculations that you have already done can you complete all of these calculations without using a calculator?

What patterns do you notice?

Why do you think that the ancient Egyptians called these numbers *magic numbers*?

$$\begin{aligned}
 (9 - 1) \div 8 &= \\
 (98 - 2) \div 8 &= \\
 (987 - 3) \div 8 &= \\
 (9876 - 4) \div 8 &= \\
 (98765 - 5) \div 8 &= \\
 (987654 - 6) \div 8 &= \\
 (9876543 - 7) \div 8 &= \\
 (98765432 - 8) \div 8 &= \\
 (987654321 - 9) \div 8 &=
 \end{aligned}$$

HELP

Start with this simpler activity:

Fill in the boxes:

$$136 \div 8 = 17 \text{ because } 136 = 80 + \square$$

$$280 \div 8 = 35 \text{ because } 272 = \square + 40$$

$$456 \div 8 = 57 \text{ because } 456 = \square + \square$$

To do the division $2760 \div 8$ by chunking
split up 2760 giving $\square + 320 + 40$.

Think how many 8's in the piles of hundreds, tens and units

$$8 \times \square + 8 \times \square + 8 \times \square$$

Explain how you filled in the boxes

Then try the challenge above again. You can do it!! Say to yourself YES I CAN.

NEXT MORE MAGIC NUMBERS

This challenge is an extension to MAGIC NUMBERS

<https://aiminghigh.aimssec.ac.za/grades-4-to-7-magic-numbers/>

Find the numbers to put in the boxes to make the calculations in LIST 1 correct.

LIST 1

$$\begin{aligned}\square \times 8 + 1 &= 9 \\ \square \times 8 + 2 &= 98 \\ \square \times 8 + 3 &= 987 \\ \square \times 8 + 4 &= 9876 \\ \square \times 8 + 5 &= 98765 \\ \square \times 8 + 6 &= 987654 \\ \square \times 8 + 7 &= 9876543 \\ \square \times 8 + 8 &= 98765432 \\ \square \times 8 + 9 &= 987654321\end{aligned}$$

LIST 2

$$\begin{aligned}(9 - 1) \div 8 &= \\ (98 - 2) \div 8 &= \\ (987 - 3) \div 8 &= \\ (9876 - 4) \div 8 &= \\ (98765 - 5) \div 8 &= \\ (987654 - 6) \div 8 &= \\ (9876543 - 7) \div 8 &= \\ (98765432 - 8) \div 8 &= \\ (987654321 - 9) \div 8 &= \end{aligned}$$

Complete the calculations in LIST 2.

What do you notice about the connections between the equations:

$$\square \times 8 + 1 = 9 \text{ etc.}$$

and the calculations:

$$(9 - 1) \div 8 = ? \text{ etc. ?}$$

What do **inverse operations** have to do with the connections between LIST 1 and LIST 2?

You have been doing algebra!

The equations $\square \times 8 + 1 = 9 \text{ etc.}$ can be written in the form

$$8x + 1 = 9 \text{ etc.}$$

where the letter x represents an unknown number and $8x$ means x multiplied by 8.

When you do algebra you are asked to solve equations. That means to find the number that the letter represents.

How would you use inverse operations to solve equations like $8x + 1 = 9$

$$8x + 2 = 98 \text{ etc. ?}$$

NOTES FOR TEACHERS

MAGIC NUMBERS SOLUTION

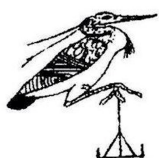
$$\begin{aligned}
 9 - 1 &= 8 \\
 98 - 2 &= 96 \\
 987 - 3 &= 984 \\
 9876 - 4 &= 9872 \\
 98765 - 5 &= 98760 \\
 987654 - 6 &= 987648 \\
 9876543 - 7 &= 9876534 \\
 98765432 - 8 &= 98765424 \\
 987654321 - 9 &= 987654312
 \end{aligned}$$

Many patterns can be observed in the solutions to these subtraction calculations. The subtractions are the first step to calculating the answers to the calculations $(9 - 1) \div 8 = ?$ etc. that are given in the Extension Question 'More Magic Numbers'.

But there are 2 concepts to be understood before proceeding with those calculations:

(1) The gold coins story is designed to help learners to understand the process of division;

(2) Learners need to understand the use of brackets in the calculations. This is a 'pre-algebra' activity.



This illustration gives the solution to the MORE MAGIC NUMBERS CHALLENGE.

$$\begin{aligned}
 1 \times 8 + 1 &= 9 \\
 12 \times 8 + 2 &= 98 \\
 123 \times 8 + 3 &= 987 \\
 1234 \times 8 + 4 &= 9876 \\
 12345 \times 8 + 5 &= 98765 \\
 123456 \times 8 + 6 &= 987654 \\
 1234567 \times 8 + 7 &= 9876543 \\
 12345678 \times 8 + 8 &= 98765432 \\
 123456789 \times 8 + 9 &= 987654321
 \end{aligned}$$

www.gold6958.wordpress.com

Why do this activity?

Discovering the pleasing patterns should give this activity an appeal for learners. They will get practice in subtraction and division in an interesting way. Moreover the activity is designed to help learners to understand three fundamental mathematical ideas: inverse operations; the process of division and how it is equivalent to 'sharing'; and how solving linear equations involves the use of inverse operations.

Learning objectives

In doing this activity students will have an opportunity to gain a better understanding of:

- (a) the processes of subtracting and dividing;
- (b) what inverse operations are and how to use them;
- (c) how to solve linear equations.

Generic competences

In doing this activity students will have an opportunity to develop the skill of interpreting, creating and visualising images to represent concepts and situations.

Diagnostic Assessment

This should take about 5–10 minutes.

Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.

$$60 \div (5 \times 2) + 3$$

A	B	C	D
10	8	9	4

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer to practice their communication skills and sort out their thinking by putting their ideas into words.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

5. As the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is C.

A. and B. Students giving these answers seem to be very poor at simple arithmetic.

D. One student explained his answer: Because $5 \times 2 = 10 + 3 = 13$ and $60 / 13 = 4$

<https://diagnosticquestions.com>

Suggestions for teaching

Start with the Diagnostic Quiz as a warm up.

9 - 1 =	Lesson starter (10 minutes) Write the subtractions on the board and ask the learners to copy them into their notebooks, to work them out and, when they have finished, to make a note of what they have noticed.
98 - 2 =	
987 - 3 =	
9876 - 4 =	
98765 - 5 =	
987654 - 6 =	
9876543 - 7 =	
98765432 - 8 =	
987654321 - 9 =	

Dividend
↓
40 ÷ 8 = 5
Divisor Quotient

Main lesson: This lesson focusses on division. Use the diagram to introduce the idea of dividing by 8.

Copy the gold coins sharing story on the board. Ask the learners to talk with their partners, or in small groups, to find the best way to use the coin sharing story to explain the division sum:

$$9872 \div 8 = 1000 + 200 + 30 + 4 = 1234.$$

After about 5 minutes have a class discussion about this.

$$\begin{aligned} (9 - 1) \div 8 &= \\ (98 - 2) \div 8 &= \\ (987 - 3) \div 8 &= \\ (9876 - 4) \div 8 &= \\ (98765 - 5) \div 8 &= \\ (987654 - 6) \div 8 &= \\ (9876543 - 7) \div 8 &= \\ (98765432 - 8) \div 8 &= \\ (987654321 - 9) \div 8 &= \end{aligned}$$

Then copy this list of calculations on the board. Tell the learners that the brackets tell you that you must do the calculation in the brackets first and ask learners to complete the calculations.

Depending on the stage that your learners are at you could allow the use of calculators or not.

$$\begin{aligned} (9 - 1) \div 8 &= 1 \\ (98 - 2) \div 8 &= 12 \\ (987 - 3) \div 8 &= 123 \\ (9876 - 4) \div 8 &= 1234 \\ (98765 - 5) \div 8 &= 12345 \\ (987654 - 6) \div 8 &= 123456 \\ (9876543 - 7) \div 8 &= 1234567 \\ (98765432 - 8) \div 8 &= 12345678 \\ (987654321 - 9) \div 8 &= 123456789 \end{aligned}$$

Final plenary to end the lesson: Write the second list on the board and ask 9 learners to come out one by one to fill in their answers line by line.

Ask learners to tell the class what they have noticed.

Summarise what they have done to help the learners to understand and to remember what they have learned about division and about the use of brackets.

Key questions

- What do you notice?
- How does splitting the dividend into separate parts help you to share it between a number of people?
- To divide a dividend by a number (divisor) how does it help to split the dividend into separate parts?
- Why do you think we have put brackets in those calculations?

Follow up

The MORE MAGIC NUMBERS challenge is an extension of the MAGIC NUMBERS activity leading to methods of solving linear equations.

<https://aiminghigh.aimssec.ac.za/years-7-11-more-magic-numbers/>

Also see Beautiful Numbers:

<https://aiminghigh.aimssec.ac.za/years-6-8-beautiful-numbers/>