AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

DETOURS (or 3 point journeys) is the theme

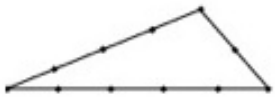
for this INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 4 to 17+

Just choose whatever seems suitable for your group of learners

The MAKING TRIANGLES activity was designed for Years 4 to 8.

MAKING TRIANGLES



Use 11 sticks of equal length to make this triangle with edge lengths 2, 4 and 5.

You might like to record this as (2, 4, 5).

How many other triangles can you make with 11 sticks?

Investigate all the triangles that you can make with different numbers of sticks.

Record your results in this table.

Number of sticks	1	2	3	4	5	6	7	8	9	10	11
Triangles											

Is it possible to make a triangle with edge lengths 2, 3 and 6?

What about 2, 3 and 4?

When can you make a triangle and when is it impossible? What patterns can you find?

To investigate this question you could use toothpicks, or paper sticks or a piece of string with equally spaced knots.

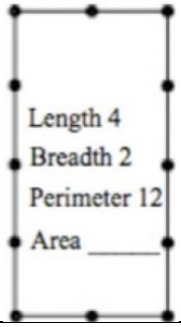
To make 4 paper sticks each 10.5 cm long, cut a piece of A4 paper in four, roll the paper with the string inside so that you can join the sticks by tying the string.

HELP

You can make a triangle with 3 sticks but not with 2 sticks or 4 sticks. Try it and explain why.

Make chains of sticks and make triangles with them. Record the triangles you find by 3 numbers for the lengths of the edges, for example the triangle shown is (2, 4, 5).
To be sure that you have found all possible cases you need to make a systematic plan.

NEXT



Wholesome Rectangles

https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/

is about exploring all the possibilities fordifferent rectangles that have the same perimeter and whole number dimensions, and then finding their areas.

Resources: paper sticks with string inside that can be tied in a loop or a collection of twigs, ot stems of plants, or toothpicks … with all the sticks the same length.

INCLUSION AND HOME LEARNING GUIDE

THEME: DETOURS (OR 3 POINT JOURNEYS AROUND TRIANGLES).

Early Years

For this activity you can use twigs from a tree, even grass, or toothpicks or matchsticks or paper sticks…
Collect some sticks and break or cut them to the same length.
You'll need help with this.

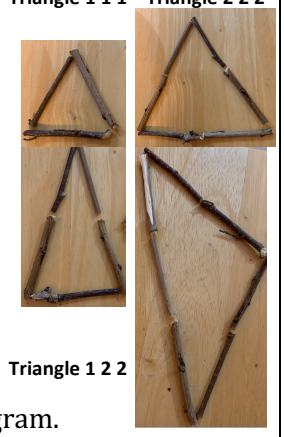
Make a triangle with 3 sticks.

Try 4 sticks and 5 sticks and 6 and 7.

Can you always make a triangle? Why or why not?

How many different triangles can you make?

Lower Primary



Triangle 1 1 1 Triangle 2 2 2

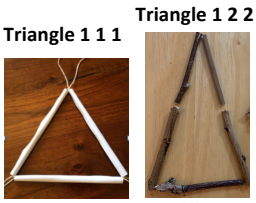
Triangle 1 2 2

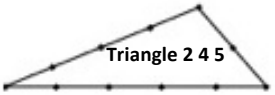
Triangle 2 2 3

For this activity you can use twigs from a tree, even grass, or toothpicks or matchsticks or paper sticks…
Work in a group if possible.
Collect some sticks and break or cut them all the same length.

1. Make a triangle with 3 sticks.
   What do you notice about your triangle?
2. Try 4 sticks. What happens why?
3. Make triangles with 5 sticks and 6 and 7.
   Give names to your triangles, label them like the triangles in the diagram.
   What do you notice about your triangles? Find the perimeter for each of your triangles in stick lengths.
4. What's the same and what's different about the triangles you have made?
5. If you start with some sticks, can you always make a triangle using them all? Why or why not?
6. Can you sometimes make 2 or more different triangles with the same number of sticks?
7. Make a record of the collection of triangles that you have made.
8. How many different triangles can you make?

Upper Primary and Lower Secondary





For this activity you can use twigs from a tree, even grass, or toothpicks or matchsticks or paper sticks…
Work in a group if possible.
Collect some sticks and break or cut them all the same length.

Investigate all the triangles that you can make with different numbers of sticks.

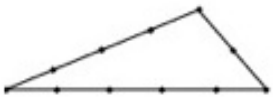
Record your results in a table.

Number of sticks	1	2	3	4	5	6	7	8	9	10	11
Triangles											

How to help the learners:

Ask learners to try to make triangles with 1 stick, and then with 2 sticks, then 3, then 4. Can they explain why it is sometimes impossible to make a triangle with a given number of sticks. They may not have an explanation. You can come back to this later.

The stringy stick loops are very suitable for investigating triangles because you can quickly change the shape while keeping the same perimeter. They are also good for holding up to show other people.



1. Use a loop or chain of 11 paper sticks. You can do this as a group activity. It's ideal for a mixed-age group. Make a triangle with edge lengths 2, 4 and 5. Record this as 2 4 5 or (2, 4, 5).
2. Then ask the learners how many other triangles they can make with 11 sticks. Ask them to make and record as many different triangles as possible that can be made with an 11-stick stringy loop or 11 twigs or other sticks.
3. Explain that this activity is about exploring the possibilities for triangles with whole number dimensions. With other lengths there are infinitely many possible triangles so here we just stick to whole numbers. In order to find all the triangles with a given perimeter learners need to **work systematically** and this is an important mathematical skill.
4. Ensure that all learners DO the activity, TALK about it and RECORD their working, that is that they engage in all three of the '**Do, Talk, Record**' actions.
5. Check that learners are recording all their findings. It is important **to record results clearly and systematically**, another important mathematical skill. The results can be recorded using diagrams and numbers to represent the edge lengths, or using a table. Depending on your group you might let learners find their own ways to record results or you might suggest using a table.
6. Finish the session with a discussion of all the results that they have found and record them systematically.
7. If you let them choose their own methods, have a discussion about the advantages of different methods of recording.

8. Discuss the rule for when it is possible to make a triangle and why it is impossible for some sets of three lengths.
9. You may like to get learners to copy notes into their workbooks like this 'Fact Box'.
10. You might line to use the Diagnostic Quiz to see how much they have learned.

Key questions

- What can you tell me about the shape you have made?
- How did you decide how to record your data?
- How do you know if you have found all the possibilities?
- Can you find any patterns?
- Can you carry on and find more results using bigger loops or using bigger numbers without using a loop?
- How would you describe the area and perimeter of your shape?
- What do you notice? What is the same? What is different?
- List all the combinations of 3 whole numbers that add up to 12 (for example 1 + 4 + 7). Which of these number triples can make the edge lengths for a triangle? Give reasons.

Fact box

Perimeter

The perimeter of a shape is the distance all the way around the edges, the total length of the boundary. For example, if a farmer puts a fence all the way around a field, of any shape then the length of fencing needed is the perimeter. For a polygon the perimeter is the sum of the lengths of the edges. Perimeter is measured in centimetres, metres or kilometres.

Area

The area is the amount of space inside the boundary of a 2-dimensional object such as a triangle, a square or a rectangle or any shape such as this squiggle.

Area is measured in square centimetres, square metres or square kilometres. There are different ways to work out the areas of different shapes including integral calculus where we imagine the area made up of infinitely many infinitely thin rectangular strips.

Triangle Inequality

For 3 lengths to make the edges of a triangle the rule is:
The sum of the lengths of any two edges is greater than the length of the third edge.

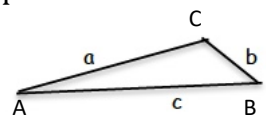
Lengths *a*, *b* and *c* make a triangle if and only if $a + b > c$, $b + c > a$ and $c + a > b$.

Upper Secondary

Detours or 3 point journeys around triangles

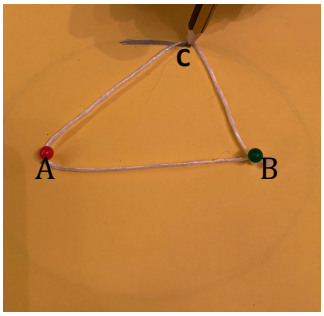
You could start with the activity described on page 1. This investigation is all about triangles with edges made up of whole number lengths. The challenge is to explain WHY some lengths can make the edges of a triangle and other lengths cannot. You should try to explain the reasons to someone else and write down your explanation.

Next consider triangles where the edges can be any lengths, not just whole number lengths. What is special about triangles where the length AB is constant and the perimeter of the triangle ABC is constant?



Think about places you can visit when travelling in straight lines **from A to B via another place C**, making a triangle.

To investigate this you need some string, 2 pins, a pencil and a board or some magazines that you can stick the pins into. Make a loop of string and place it over the 2 pins. Keeping the string taught so that the length of the string stays the same, and therefore the perimeter of the triangle formed by the string stays the same, what can you say about the path, or locus, that your pencil will take? Try this.



The diagram shows one possible triangle. There can be infinitely many triangles but the pencil (on the moving point we call C) is not free to move anywhere but rather it moves on a pre-determined path.

Try this with string loops of different lengths and the pins closer or wider apart. What can you deduce about the path of he moving point C?

Why do this activity?

This activity reinforces the understanding of perimeter and it is accessible to all abilities. Weaker learners can be successful in making triangles and counting the sticks. Some of the ablest learners may discover the triangle inequality through Activity 1.

Rather than just hearing about it from somebody else, discovering the triangle inequality should be a satisfying experience and boost learners' confidence. Modelling the mathematical concept is powerful because it is intuitively obvious from using the sticks that certain triangles cannot be made but it is quite difficult to think about in the abstract.

The **general teaching strategy** here is **modelling mathematical concepts using manipulatives** so that learners can easily change the shapes and explore the possible configurations. Handling the manipulatives helps learners to visualise the mathematical shapes and their properties. The objective is for learners to progress to being able to visualise the shapes without using the concrete materials and to be able to work from the numbers and the properties of the shapes where different numbers are involved.

Learning objectives

In doing this activity students will have an opportunity to

- gain a deeper understanding of the concept of perimeter;
- learn about the ellipse, one of the conic sections.

Generic competences

In doing this activity students will have an opportunity to develop problem solving, mathematical thinking and systematic working skills.

SOLUTION

With 11 sticks you can make 4 different triangles as shown in the last column of the table below.

Number of sticks	1	2	3	4	5	6	7	8	9	10	11
Triangles	None	None	(1,1,1)	None	(1,2,2)	(2,2,2)	(1,3,3) (2,2,3)	(2,3,3)	(1,4,4) (2,3,4) (3,3,3)	(2,4,4) (3,3,4)	(1,5,5) (2,4,5) (3,3,5) (3,4,4)

Even though 2 + 3 + 6 = 11 it is not possible to make a (2, 3, 6) triangle because 2 + 3 < 6.

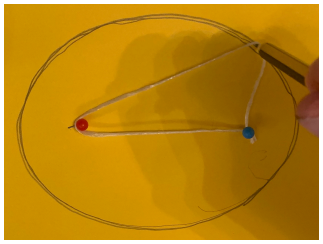
For 3 lengths to give a triangle the rule is that the sum of the lengths of the any two edges must be greater than the length of the third edge.

For lengths a, b and c to make a triangle: a + b > c, b + c > a and c + a > b.

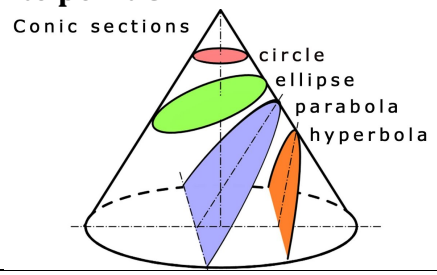
This is called **the triangle inequality**. Check that it works for your triangles.

A (2, 3, 4) triangle with a perimeter of 9 units can be made.

Constant distance AC + CB from A to B via a detour to point C



The moving point C lies on an ellipse. A and B are called focal points of the ellipse. The sum of the distances from the moving point C to the foci is constant.



DIAGNOSTIC ASSESSMENT This should be used after the lesson.

Show this question and say:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Which of the following sets of 3 lengths cannot make the edges of a triangle?

A. 3, 4, 6

B. 3, 4, 8

C. 5, 12, 13

D. 4, 5, 6

1.Notice how the learners respond. Ask them to explain why they gave their answer. DO NOT say whether it is right or wrong, simply thank them for the answer.
2.It is important for learners to explain the reason for their answer so that, by putting their thoughts into words, they gain a better understanding and develop communication skills.
3.With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: B. Join the edges of lengths 3 and 4 units.
It's impossible to make the third edge of the triangle longer than 7 units.
https://diagnosticquestions.com

Follow up

Wholesome Rectangles
https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/
is about exploring the possibilities for rectangles with whole number dimensions and finding their perimeters and areas.

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13