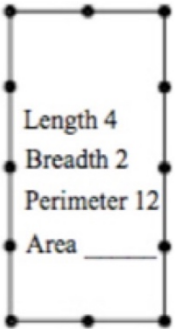
WHOLESOME RECTANGLES



This activity is about rectangles with whole number dimensions. With other lengths and breadths there are infinitely many possible rectangles so here we just stick to whole numbers.

What do you know about rectangles? What is the smallest rectangle you can make with edge lengths that are whole numbers?

How many rectangles can you make with perimeter 12 units?

You might like to experiment by making rectangles with chains of paper sticks or with toothpicks.

Draw your rectangles on squared paper and record their length, breadth, perimeter and area.

Choose another perimeter and find all possible rectangles with that same perimeter. You could make a table to keep a record of all your results.

Can you find other rectangles with the same perimeter and different areas?

For rectangles with the same perimeter, what do you notice about the rectangle with the biggest area?

Can you find rectangles with the same area and different perimeters?

What else do you notice? Can you find connections with factors and multiples?

HELP

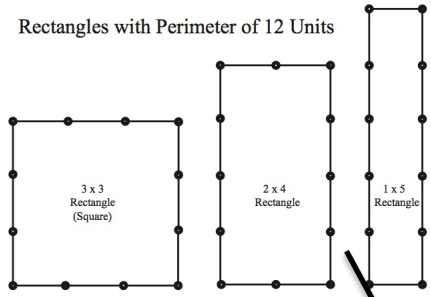
If you draw the rectangles on squared paper, you can find the perimeter by counting the units of length, and you can find the area by counting the squares each of which has area 1 square unit.

NEXT

Can you find all the rectangles with perimeter 22 without using toothpicks or first drawing the rectangles?

NOTES FOR TEACHERS

SOLUTION



The smallest rectangle of this type is the 1 by 1 square of area 1 and perimeter 4.
A rectangle has:
4 edges
4 right angles
Opposite edges equal in length
Opposite edges parallel.
There are 3 rectangles with perimeter 12 units.

This is one way to record results. Each box gives all the rectangles with the same perimeter.

Breadth	1	1	1	2	1	2	1	2	3	1	2	3	1	2	3	4	1	2	3	4	
Length	1	2	3	2	4	3	5	4	3	6	5	4	7	6	5	4	8	7	6	5	
Perimeter	4	6	8	8	10	10	12	12	12	14	14	14	16	16	16	16	18	18	18	18	20
Area	1	2	3	4	4	6	5	8	9	6	10	12	7	12	15	16	8	14	18	20	

The perimeter of all rectangles of this type are **even** and equal to or greater than 4. An odd perimeter cannot arise because the length and breadth are whole numbers and the perimeter is 2(length + breadth).

We find rectangles with the same perimeter and different areas **by finding different pairs of numbers that add up to the same number (the semi-perimeter)**. Such rectangles are shown in the cells of the table above.

For rectangles with the same perimeter the rectangle with the biggest area is the closest to a square. If the perimeter p is a multiple of 4 then the biggest area is $(\frac{1}{4}p)^2$.

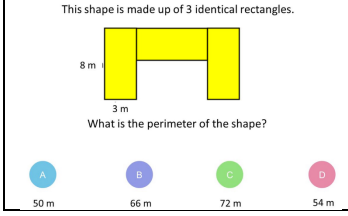
If p is not a multiple of 4 then the biggest area is $(\frac{1}{4}p + \frac{1}{2})(\frac{1}{4}p - \frac{1}{2}) = (p^2 - 4)/16$.

We find rectangles with the same area and different perimeters **by finding different pairs of factors of the same number (the area)**. For example the 3 by 4, the 2 by 6 and the 1 by 12 rectangles all have area 12.

The length and breadth of a rectangle are **factors** of the area and the area is a **multiple** of the length and breadth.

Diagnostic Assessment

This should take about 5–10 minutes.
Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is D
A. This may be a mistake in adding the lengths.
B. This is the total perimeter of the 3 rectangles separately ignoring that the edges that touch are counted twice.
C. This shows confusion between perimeter and area as the area is 72 m².

https://diagnosticquestions.com

Why do this activity?

This activity provides an experience of modelling mathematical concepts. Learners will revise the properties of rectangles and squares. Learners will find it a much more interesting way to practice working out perimeters and areas of rectangles, and to learn the difference between perimeter and area, than just completing practice exercises. In order to find all the rectangles with a given perimeter, and then all the rectangles with a given area, learners need to think mathematically and to work systematically, both important skills.

Learning objectives

In doing this activity students will have an opportunity to:

- get to know facts about the area and perimeter of rectangles, and about factors and multiples;
- gain a deeper understanding of perimeters and areas of rectangles by investigating and extending whole number patterns in geometry;
- be able to create polygons, to record observations systematically and analyse the results to find all possible cases;
- appreciate the power of observing patterns and representing results in physical, symbolic, diagrammatic and tabular forms;
- have experienced using manipulatives and diagrams in order to aid visualisation and then progressing to apply the same reasoning without depending on manipulatives and diagrams.

Generic competences

In doing this activity students will have an opportunity to:

- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- **persevere and work systematically** to investigate all possible cases.

Suggestions for teaching

Ask the learners "**What do you know about rectangles?"** They should be able to give you all the properties so write the properties that they tell you on the board and keep asking **"What else do you know?"** until you have them all.

Tell the class that in this lesson they are only going to work on rectangles with whole number lengths and breadths. If you have prepared a chain of paper sticks, or a knotted length of rope or string, you might make a 4 by 2 rectangle with it and get two learners to hold it up to show the class. Draw the 4 by 2 rectangle on the board and ask the learners what area does it have and what perimeter?

Ask learners "What is the smallest rectangle of this type?" If nobody suggests the unit square it may be because they do not realise that a square is a rectangle, so keep asking "Can you find a smaller one?" Ask the learners the following questions to which the answer is yes every time, proving that a square is a rectangle:

"Does a square have 4 edges?" "Does a square have 4 right angles?"
"Does a square have opposite edges equal in length?"
"Does a square have opposite edges parallel?"
"Is a square a rectangle?"

Ask the learners to find other rectangles that have perimeter 12 units. Learners could make

their own chains of paper sticks from scrap paper. If available give squared paper out to each pair (or group) of learners. Ask the learners to make rectangles, to draw them on squared paper or in their notebooks and to record the length, breadth, perimeter and area in one big table for all the rectangles that they find. You might start a table on the board to help the learners to construct their own tables of results.

Then write the remaining questions on the board or give out worksheets copied from page 1. Ask the learners to talk in pairs or groups about the properties of the rectangles they find and what they notice about area, factors and multiples.

Finally have a whole class discussion in which the learners tell you what they have noticed about factors and multiples. Then ask learners to give all the rectangles with perimeters 4, 6, 8, 10, 12, 14, 16, 18. Fill in the table on the board and tell learners to check and extend the tables that they have recorded in their notebooks.

If there is not enough time in the lesson to complete the table you could ask the learners to complete the task as homework and (if time) to find all the rectangles with a perimeter of 20 units.

Key questions

- Have you found all the possibilities? How do you know?
- What can you tell me about the shape you have made?
- How would you describe the area and perimeter of your shape?
- Can you find any patterns?
- What do you notice?

Follow up

Same volume https://aiminghigh.aimssec.ac.za/years-6-to-8-same-volume/
Factors and Multiples Game
https://aiminghigh.aimssec.ac.za/years-6-12-factors-and-multiples-game/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. Note: For resources for teaching A level mathematics see https://nrich.maths.org/12339 New material will be added here for Year 13 (UK) and Secondary 6 (East Africa) that is beyond the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6