TWO ACES

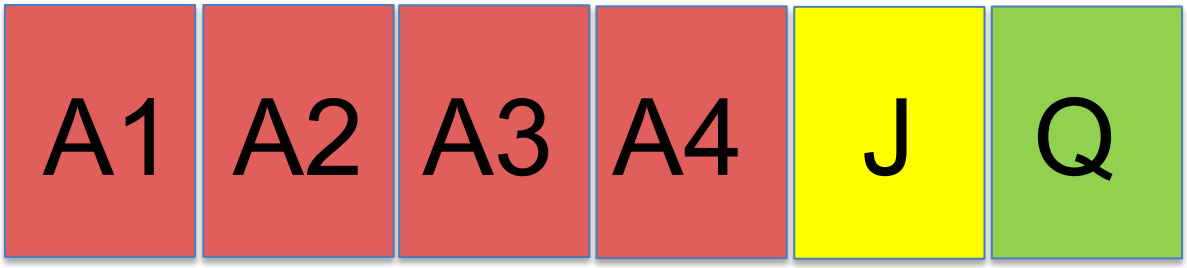
For this activity you need six cards, four Aces and the Jack of hearts and Queen of spades. If playing cards are not available, then make some coloured cards instead. (See the Two Red Cards version below)

You pick 2 cards at random and tell me "I have an Ace". What is the probability that you have two aces?

Next suppose I pick 2 of the 6 cards at random and tell you "I have the Ace of spades". What is the probability that I have two Aces?

Are these two probabilities of two Aces the same? Explain your answer.

TWO RED CARDS



We have six cards, four red cards and one yellow card and one green card.

You pick 2 cards at random and tell me "I have a red card". What is the probability that you have two red cards?

Next suppose I pick 2 of the 6 cards at random and tell you "I have the A1 card". What is the probability that I have two red cards?

Are these two probabilities of two red cards the same? Explain your answer.

HELP

Try to answer questions (1) and (2). Only study the answer below if you can't do it yourself. If possible, discuss it with someone else and help each other. When you understand this family example then answer questions (3) to (9) below to solve the Two Aces problem.

(1) You meet a mother who tells you "I have 2 children" and you know one of them is a boy. What is the probability that her other child is a boy?
(2) Now suppose she tells you "I have 2 children and the oldest is a boy," What is the probability that the other one is a boy?

Case (1) The possible families are BB, BG, GB so the probability that the other child is a boy is $\frac{1}{3}$.

Case (2) The possible families are BB, BG so the probability that the other child is a boy is $\frac{1}{2}$.

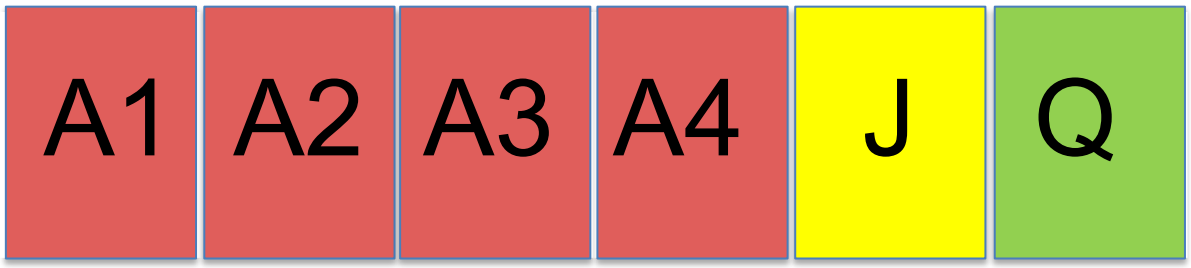
(3) How many different pairs of cards are there if you choose two cards from the six cards? Make a list of all the possibilities.
(4) How many of those pairs contain one or more Aces (red cards)?
(5) How many of those pairs have 2 Aces (2 red cards)?
(6) If you have already picked 2 cards and one is an Ace what is the probability that both are Aces?
(7) How many pairs contain the Ace of spades (A1 card)?
(8) If you have already picked 2 cards and one is the Ace of spades (A1) what is the probability that both are Aces?
(9) Are the answers to questions 2 and 6 the same? If not, why not?

NEXT

(1) You meet a mother who tells you "I have 2 children" and you know one of them is a boy. What is the probability that her other child is a boy?
(2) Now suppose she tells you "I have 2 children and the oldest is a boy," What is the probability that the other one is a boy?

Make up a similar problem to this for a family of 3 children.

Cut out these cards or make your own set.



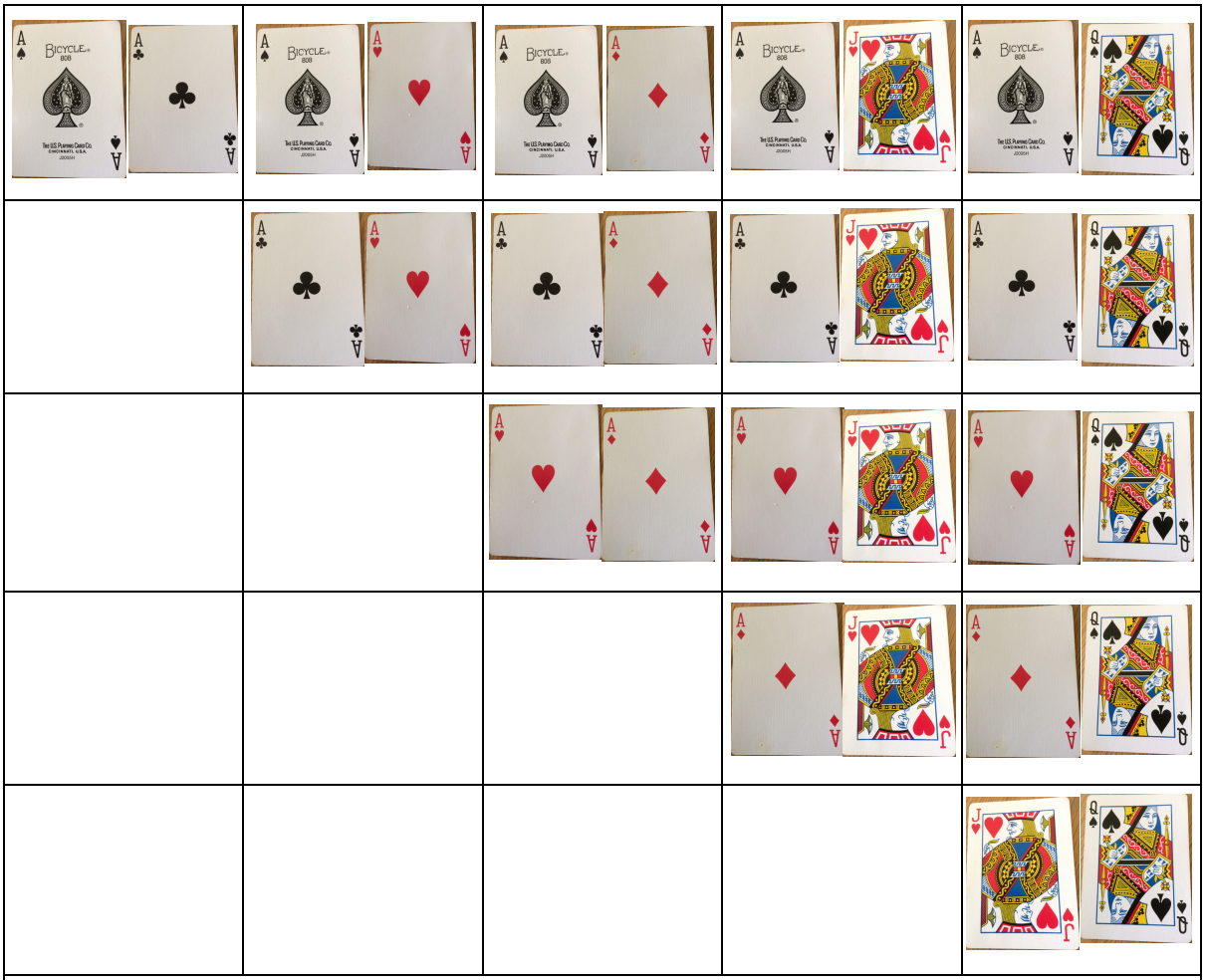
NOTES FOR TEACHERS

SOLUTION

No, the probabilities are not the same.

There are 15 possible pairs of cards as shown below. If you say you have an Ace then you have one of 14 pairs of cards, and 6 pairs contain 2 Aces, so the probability that you have 2 Aces is 6/14 = 3/7 = 0.429 (to 3 decimal places.

If you say that you have the Ace of spades then you have one of 5 possible pairs of cards, and 3 pairs contain 2 Aces, so the probability that you have 2 Aces is 3/5 = 0.6



Notice that the number of pairs is the TRIANGLE NUMBER

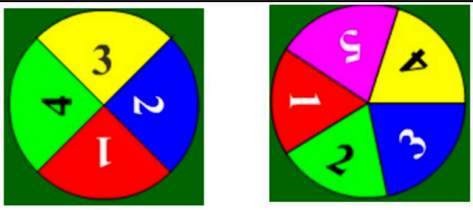
$$1 + 2 + 3 + 4 + 5 = \tfrac{1}{2}(6 \times 5) = 15$$

Each of the 6 cards can be paired with 5 other cards making 6 × 5 = 30 pairings but this counts each pair twice so the number of pairs is 15.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



You spin the above fair spinners twice

What is the probability of getting a total of 2 when you add the scores together?

A $\frac{1}{20}$ B $\frac{2}{9}$ C $\frac{4}{9}$ D $\frac{2}{20}$

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Again ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A. is the correct answer. The probability of a 1 with the 4-sector spinner is ¼. In those cases only 1/5th of the spins with the 5-sector spinner will be 1s. So the probability of two 1s is 1/4 × 1/5 = 1/20. Another way to understand this is that there are 20 possible outcomes and only one of them is a double 1 so the probability is 1/20.

Common Misconceptions

B. Learners may be using this incorrect reasoning "There are 9 chances of landing a number. There are two 2's. The probability is 2/9"

C. Learners may be using this incorrect reasoning "1/4 + 1/5 = 0.45 = 4/9"

D. Learners may reason correctly that there are 20 possible outcomes but use 2 as the number of ways of getting a total of 2 whereas there is only one possible way to get 2, which is the total when you add the two spinners together which is 1 + 1."

https://diagnosticquestions.com

Why do this activity?

This activity appears to be straightforward. It requires listing and counting the number of different pairs (combinations) of two objects chosen from six distinct objects, and then working out two probabilities.

However, the answer may be a little surprising as it might be expected that the probability of two Aces is the same in each case. But this is not so, and the activity shows learners that they need to think clearly about the information given and not jump to conclusions without justifying the answer that they give.

This is an example of conditional probability that can be found simply by counting cases and it does not require the use of a conditional probability formula.

Learning objectives

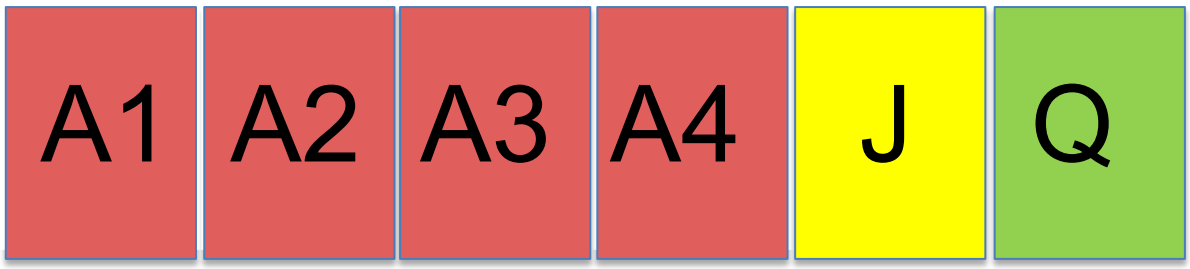
In doing this activity students will have an opportunity to:

- practice reading comprehension, that is interpreting information given
- count the number of possibilities, and to use this information to work out probabilities.
- Develop problem solving skills.

Generic competences

In doing this activity students will have an opportunity to interpret information, apply mathematical reasoning and **solve problems**.

Suggestions for teaching



Organise the class so that the learners work in pairs or small groups and give each group a set of cards. In order that learners get **practice in reading question**s, and thinking for themselves how to tackle problems, simply write the question on the board and ask the learners to find the answers.

If the learners start by working in pairs, after several of the pairs have found answers, you could ask the learners to work in fours with the pair behind or in front of them. Ask them to compare answers and to explain their reasons for giving those answers. If the pairs have not found an answer, or have found different answers, then they should try to explain what they have done, and why, and help each other to agree on one of the answers.

Observe what the learners are doing and if they are not making progress ask one of the key questions. It is important for learners to understand that you will not tell them how to solve problems, but you are asking questions to help and guide the, to the answers.

When many of the learners have found answers ask for a vote as to who thinks the two probabilities are different and who thinks they are the same. Record the number of votes cast each way.

Then ask pairs of learners to come to the board and explain how they found their answers. Ask different learners to give explanations. Draw the 15 possibilities on the board, go over the answers and summarize what has been learned.

Key questions

(1) If you choose two cards from the six cards, how many different pairs of cards are there? Make a list of all these possible outcomes.
(2) How many of the pairs contain one or more Aces (red cards)?
(3) How many of the pairs have 2 Aces (2 red cards)?
(4) If you have already picked 2 cards and one is an Ace (red card) what is the probability that both are Aces?
(5) How many pairs contain the Ace of spades (A1 card)?
(6) If you have already picked 2 cards and one is the Ace of spades (A1) what is the probability that both are Aces?
(7) Are the answers to questions 2 and 6 the same? If not, why not?

Follow up

Nines and Tens https://aiminghigh.aimssec.ac.za/years-7-10-nines-and-tens/

At Least One https://aiminghigh.aimssec.ac.za/years-8-10-at-least-one/

In the Bag https://aiminghigh.aimssec.ac.za/years-9-12-in-the-bag/

If this the that https://aiminghigh.aimssec.ac.za/years-10-12-if-this-then-that/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13