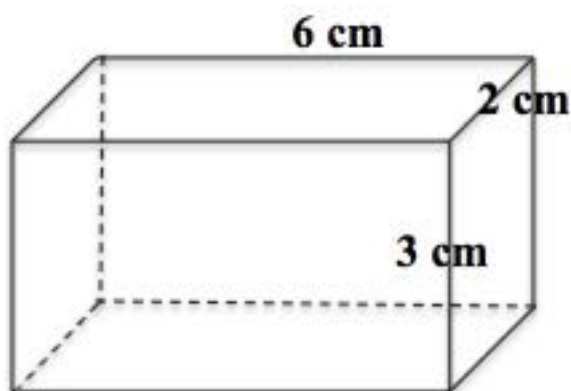


SAME VOLUME

How many cuboids with whole number dimensions have volume 36 cubic cm?



What is the volume of this cuboid?

How many other cuboids with whole number dimensions have the same volume?

Do any of them have the same surface area?

Which has the biggest surface area? Which has the smallest?

How many have a square cross section?

SOLUTION

There are 8 cuboids with volume 36 cubic centimetres. To find them all you need to list all the factors of 36 and then to find all possible sets of 3 factors that multiplied together give 36

DIMENSIONS	SURFACE AREA	
$1 \times 1 \times 36$	$2 + 4 \times 36 = 146$	Square cross section Biggest surface area
$1 \times 2 \times 18$	$4 + 36 + 72 = 112$	
$1 \times 3 \times 12$	$6 + 24 + 72 = 102$	
$1 \times 4 \times 9$	$8 + 18 + 72 = 98$	
$1 \times 6 \times 6$	$12 + 12 + 72 = 96$	
$2 \times 2 \times 9$	$8 + 4 \times 18 = 80$	Square cross section
$2 \times 3 \times 6$	$12 + 24 + 36 = 72$	
$3 \times 3 \times 4$	$18 + 4 \times 12 = 66$	Square cross section Smallest surface area

NOTES FOR TEACHERS

Why do this activity?

This activity will give learners practice in finding factors and in calculating volumes and surface area within a rich problem solving context. To be sure of finding all the solutions learners will experience the necessity of working systematically, recording the values that have been tried and avoiding repeating calculations.

Possible approach

To start the lesson you could show the learners a cuboid and review what learners know about the properties of cuboids by asking questions.



You might use a box thrown out from your kitchen that you have turned inside out to make a blank box with no writing or pictures on it - see the HOW TO TURN A BOX INSIDE OUT (page 4).

Using showboards is a good way for teachers to see how much every learner in the class knows, and not only those learners who put up their hands or who you call on to answer questions. This enables the teacher to do formative (diagnostic) assessment without it appearing to the learners like a test.



To prompt answers and discussion about cuboids ask:

“Tell me about the faces of a cuboid”

“Tell me about the edges of a cuboid”

“Tell me about the vertices of a cuboid”

“How would you work out the ‘*VOLUME*’ of a cuboid?”

“How would you work out the area of this face”? ... this one... and this one...

“What do you think the words ‘*SURFACE AREA*’ mean?”

“How would you work out the *TOTAL SURFACE AREA*?”

Main part of lesson: Then either write the question on the board or give copies to learners that you have made from page 1. Ask the learners to work out the volume of a 6 cm by 3 cm by 2 cm cuboid and, when everyone understands how to arrive at the answer 36 cubic centimetres, then they can continue to find other cuboids with whole number dimensions have the same volume and answer the rest of the questions.

Most learners will start by taking 3 numbers and multiplying them together hoping that the product is 36 (trial and error). If they try 5 or 8 for example they will find that the product is not 36. A good teacher will observe how the learners are searching and only intervene when the learner seems to be stuck. At that point you could ask questions like “why did that *not* work?” and, if the learner does not spot that they are trying numbers that are not factors of 36, the teacher can ask “does 5 divide into 36 exactly?” and “what do you call a number that divides exactly into another number?” In this way learners will learn from their mistakes and also begin to think along the right lines.

Ask learners to keep a record of what they tried that didn't work (what went wrong) as well as things that did work. It is good for learners to experiment with their own ways of recording.

When learners have found several solutions you can make a table on the board and have a whole class sharing of results. As learners give the solutions they have found you might enter the results in a systematic order as in the solution above but leaving gaps until the solution to fill that gap has been suggested by a learner. If there are still gaps then you could tell the learners that there are more solutions that they can try to find as well answering the questions about surface area. Try to ensure that learners are calculating surface area correctly.

To end the lesson: you can ask the class if anyone has found any more cuboids with volume 36 cm^3 . Learners could check their results and see if they can, between them, find all possible solutions and explain how they know they have found them all. If time they could also discuss the different ways that different learners have recorded their results and the advantages and disadvantages of the different methods of recording.

Key questions

Why did that not work?

Does that number divide into 36 exactly?

What do you notice about the numbers 6, 3 and 2 and the number 36?

What do you call a number that divides exactly into another number?"

Do factors and multiples have anything to do with this work on volumes? Can you explain that?

Can you record your answers so that it is easy to see what they are and what you have done?

Are you sure that there are no more solutions?

How do you know you have found them all?

Possible extension

The main extension activity could focus on the convincing argument that all solutions have been found. Once this has been answered, you might like to consider these extensions:

- Express the method for calculating surface area algebraically.
- CUBOIDS <https://aiminghigh.aimssec.ac.za/grade-9-or-10-cuboids/>

Possible support

Most learners will start by using trial and error as discussed above. Give them some time to try to find solutions. What learners need to do is to work out all the combinations of 3 factors of 36 that multiplied together give the answer 36. If they are really struggling you could ask them “What do you notice about the numbers 6, 3 and 2 and the number 36?” and lead them to tell you that 6, 3 and 2 divide exactly into 36. You can ask “do you think you could find more solutions like that one?”. If after a while they are still stuck you could suggest that they make a list of the factors of 36 and see if that helps.

HOW TO TURN A BOX INSIDE OUT



Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.

	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6