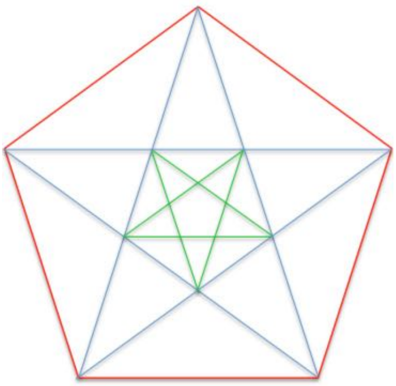
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

PENTAPATTERN



What do you notice about this diagram?

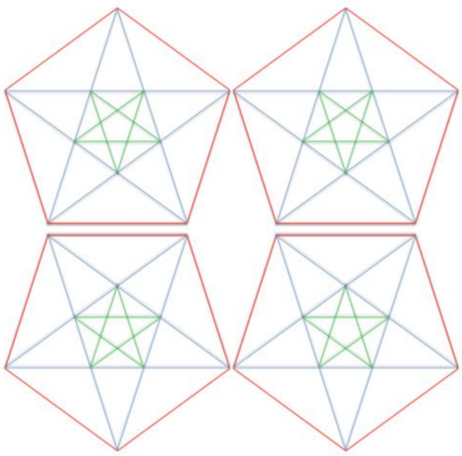
Draw the diagram accurately starting with a regular pentagon.

Find the angles in the diagram.

Label 4 sets of congruent triangles. Which triangles are similar?

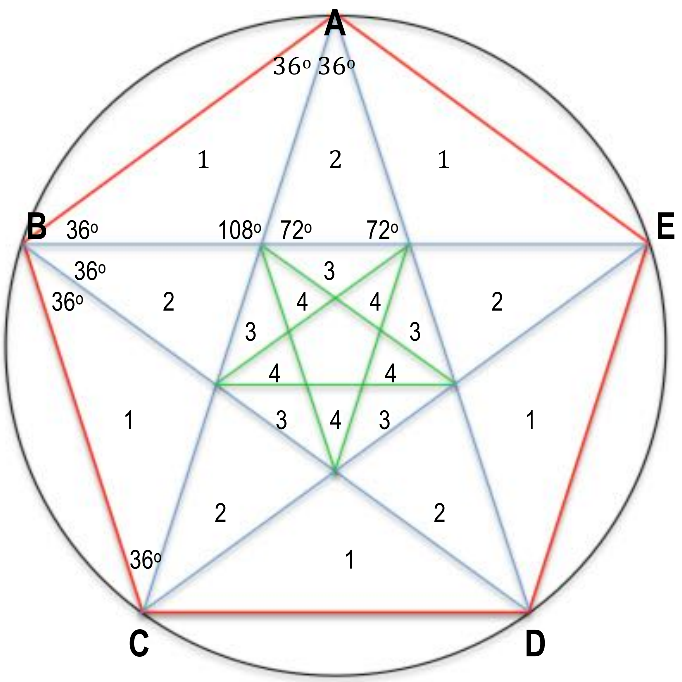
Describe the symmetries of the shape (that is the transformations that map the shape onto itself).

Describe the symmetries in the design below.



Make up your own design using pentagons and a similar construction.

SOLUTION



∠ABC = 108° (angle in regular pentagon).
ΔABC is isosceles because AB = BC
so ∠BAC = ∠BCA = 36°.
Similarly ∠ABE = ∠AEB = ∠DAE = ∠EDA
∠ECD= ∠CED = ∠CDB= ∠CBD = 36°.

It follows that all the other angles are 36°, 72° or 108° using the facts that ∠ABC and the other interior angles of the pentagon are 108°, the angles of a triangle add up to 180° and the angles on a straight line add up to 180°.

The triangles labelled 1 are all congruent and similar to the set of congruent triangles labelled 3.

The triangles labelled 2 are all congruent and similar to the set of congruent triangles labelled 4.

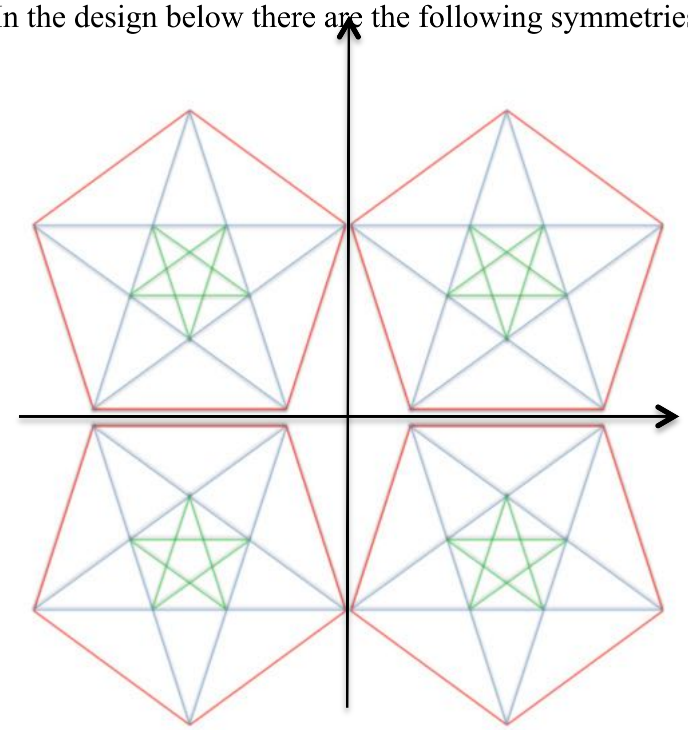
The diagram has rotational symmetry of order 5 about the centre (72°, 144°, 216°, 288°, 360°).

It has axes of reflection (mirror lines) through A and the midpoint of CD, through B and the midpoint of DE, through C and the midpoint of EA, through D and the midpoint of AB and through E and the midpoint of BC.

The green pentagram star and the blue pentagram star are enlargements of each other.

The red pentagon and the small green pentagon in the centre of the diagram are enlargements of each other.

In the design below there are the following symmetries:



1. Reflection in the y-axis
2. Reflection in the x axis
3. Rotation of order 2 (180^o) about the origin.

In addition the two pentagons in the left can be mapped to correspond to the two pentagons on the right by a translation in the x-direction.

Note: The design does not have rotational symmetry of order 4 (90^o).

NOTES FOR TEACHERS

Why do this activity?

This diagram, with a star formed by drawing diagonals in a regular pentagon, has been studied and used as a religious symbol for thousands of years, for example in the Babylonian, Egyptian, Greek and Chinese civilisations. It now features in the national flags of Ethiopia and Morocco.

This could be a good revision activity as learners need to practice and use:

1. accurate geometrical construction (measuring angles and length),
2. geometric properties of triangles and regular polygons (angle properties, similarity, congruence) and
3. geometric transformations.

The activity helps to familiarise learners with the properties of the regular pentagon and inscribed pentagram star and connects with work using trigonometry and solving equations to explore the properties of the shape and to discover that the ratio of the length of the diagonal to the length of the edge of the pentagon, and many other ratios between lengths, give the golden ratio.

This diagram has fractal properties – the pattern can be continued indefinitely inside and outside the original pentagon producing smaller and larger copies of itself.

Intended learning outcomes

1. Practice in using a protractor and ruler to make an accurate geometrical drawing.
2. Practice in finding angles in a diagram using geometric properties of triangles.
3. Practice in finding recognising similar and congruent triangles.
4. Practice in naming and describing symmetries and geometric transformations.

Suggestions for teaching

You might start the lesson by reviewing the fact that the interior angles of a regular pentagon are 108°. You might ask 5 learners to come to the front and hold hands with their arms stretched out so that their arms form the edges of a pentagon. (Alternatively use string or rope and ask learners to stand inside it holding the string with their fingers to make a pentagon). Then ask another learner to walk around the people-pentagon, turning at each corner, until he gets back to his starting point facing in the same direction as he started. Ask the class "what total angle has he turned through?", "how many turns?", "if the pentagon is regular (equal sides, equal angles) what angle was each turn?" (Answer 72°)

This is then a question that you could simply give to the learners to do for themselves, either individually or in pairs. Finally have a class discussion in which the learners share their answers, reasons and observations with the whole class.

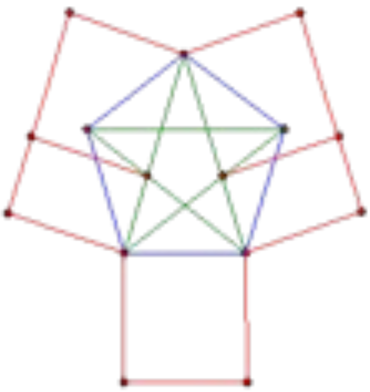
Finally summarise what has been learned.

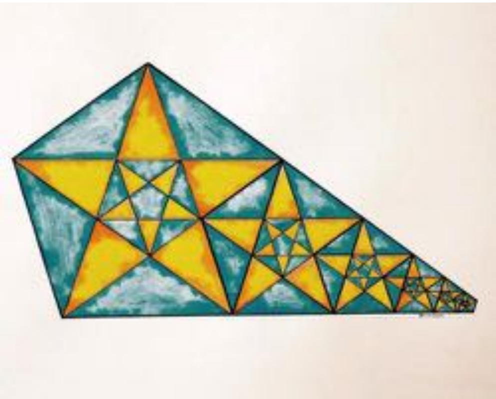
Key questions

What do you notice? Can you explain that?
Which triangles are congruent to each other? Why?
Which triangles are similar to each other? Why?
Can you see any reflections? Where is the axis of reflection (mirror line)?
Can you see any rotational symmetry? Where is the centre of the rotation?
What angles can the shape be rotated by to map onto itself? What is the order of rotational symmetry?
Can one part of the diagram be translated to map onto a corresponding part? How?

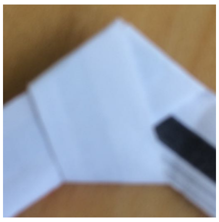
Possible extension

Learners might make their own patterns using similar constructions.
You could ask groups of learners to look for information about the regular pentagon and star pentagram on the internet and to make posters about it.
You could ask learners to describe and comment on one of the following diagrams:





Possible support



Learners can easily make a regular pentagon by tying a simple knot in a strip of scrap paper and flattening it out as shown in the diagrams.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6