EPIDEMIC

Twenty per cent of the inhabitants of a city have been inoculated against a certain disease. A city is hit by an epidemic. The chance of infection amongst those inoculated is 10% but amongst the rest it is 75%.

Copy and fill in the contingency table and the Venn diagram below and use them to answer the questions.

1. a. In the contingency table, split the 20% who are inoculated into the percentages who get infected and the percentages who are not affected (remain immune).

b. Split the 80% who are not inoculated into three quarters who get infected and one quarter who are not infected (who remain immune).

c. Fill in the totals in the right hand column.

2. What proportion are infected?

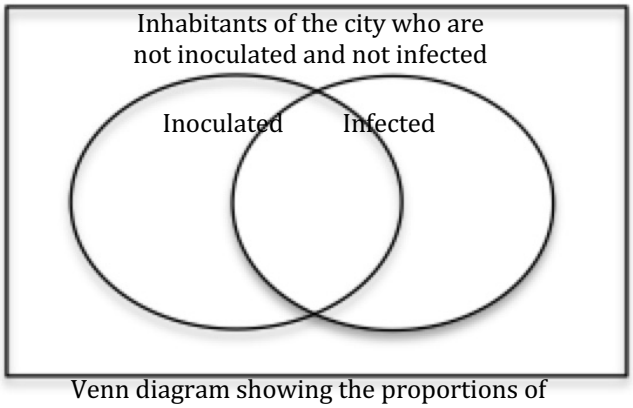
3. If a man is chosen at random and found to be infected, what is the chance of his having been inoculated?

	Inoculated	Not inoculated	Totals
Infected			
Not infected			
Totals	20%	80%	100%

Note: fractions, decimals and percentages are 3 alternative ways of writing the same number.

This example refers to proportions of the population not to actual numbers.

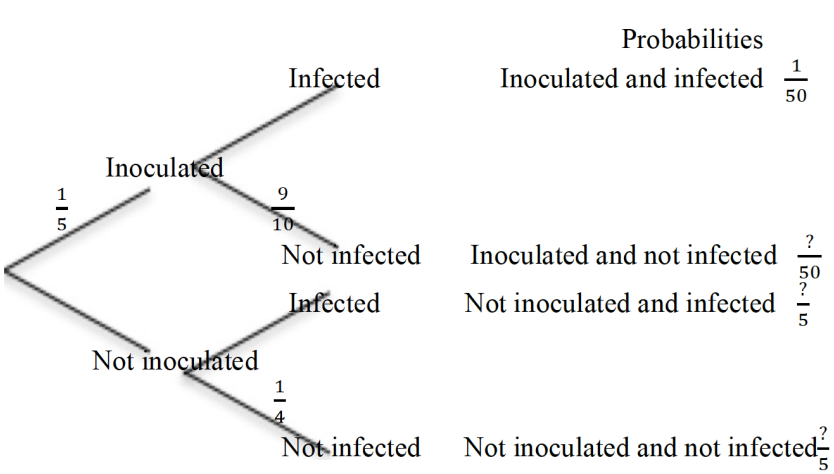
Sometimes fractions are convenient, but we often use decimals for calculations.



Venn diagram showing the proportions of inoculated or infected people in the city.

4. Complete this Venn diagram, and also complete the tree diagram below.

The contingency table, Venn diagram and tree diagram give the same information in different ways.



Fill in the given percentages as fractions, and to show the probabilities of an individual chosen at random from the population being:
(i) inoculated or not inoculated, and
(ii) being infected or not infected.

5. The chance of an infected person having been inoculated is $\frac{1}{50}/\frac{31}{50} = \frac{1}{31}$.

What is the chance of a person not infected having been inoculated?

HELP

If possible work in a group where different people bring different experience, skills and knowledge to contribute to the group discussion. Talk to other people about this problem but aim for everyone to help everyone else.

There is much talk in the media about the corona virus. Some of what you hear is inaccurate and misleading. For example, when people speak of 'safe' they should often say 'lower risk' because going into places where there are other people is not perfectly safe.

When people speak of medical test results you may believe the results are accurate. Test are not always accurate. They can show a person does not have the virus when they do have it or that a person is free of the virus when they are not (called false-positives and false-negatives). Nothing is certain except that what happens in the future will depend on probability.

NEXT

Thinking of your own gender and height which of the 4 boxes in the contingency table are you in?

Can you explain why you are in that box and not in any of the other three boxes?

	Under 175 cm in height	**175 cm in height or taller**	**Totals**
Male	Number of males who are under 175 cm in height	Number of males who are 175 cm in height or taller	
Female	Number of females who are under 175 cm in height	Number of females who are 175 cm in height or taller	
Totals			

Thinking about complementary sets, would you say that the sets of males and females are complementary sets? Why? Would you say that the sets of people under 175 cm in height and of people 175 cm in height or taller are complementary sets? Why?

Either using real data from a class in your school or some made up data, make up a contingency table using these attributes. When you have found the totals of males and females what relative frequencies can you calculate and what are they?

When you have found the total number of people under 175 cm in height and the total number of people 175 cm or taller what relative frequencies can you calculate and what are they?

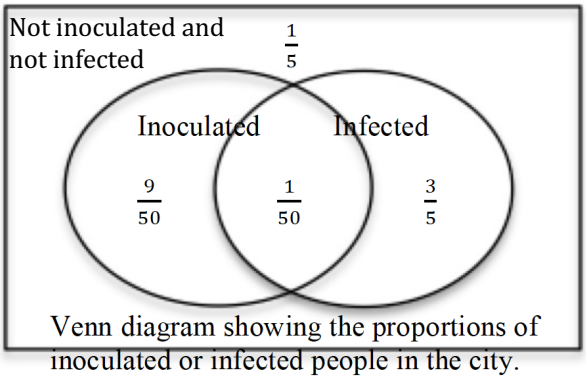
Suppose you recorded this data from a Year 12 class, would these relative frequencies be the same as if you took a random sample of 100 learners from your **whole school**? Explain your answer.

Draw a Venn diagram to represent the data in your contingency table**.**

NOTES FOR TEACHERS

SOLUTION

	Inoculated	Not inoculated	Totals
Infected	2%	60%	62%
Not infected	18%	20%	38%
Totals	20%	80%	100%

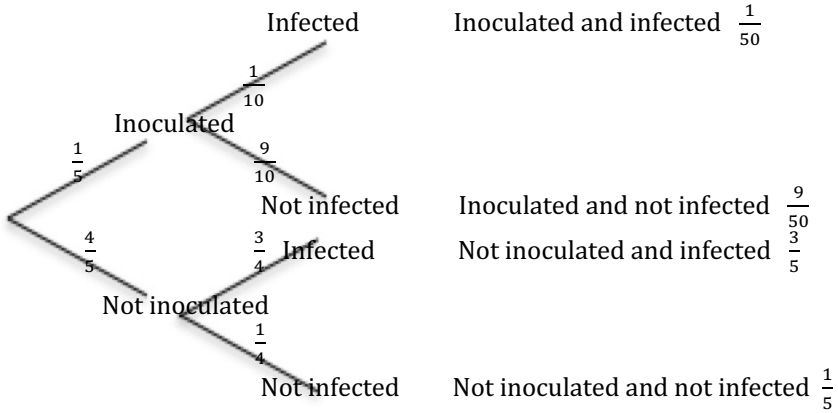


Venn diagram showing the proportions of inoculated or infected people in the city.

2. 62% are infected. This can also be given as 0.62 or $\frac{31}{50}$.

3. If a man is chosen at random and found to be infected, the chance of his having been inoculated is $\frac{2}{62} = \frac{1}{31}$ = 0.03 to 2 decimal places.

4.



5. The chance of a person not infected having been inoculated is $\frac{9}{50}/\frac{19}{50} = \frac{9}{19}$ = 0.47 to 2 decimal places.

Why do this activity?

This activity involves learners in creating and interpreting different representations of probability. Learners will review their knowledge and understanding of the concepts and of the language of probability by connecting the contingency table, Venn diagram and tree diagram representations of a set of data. Working on this example should help learners to understand more about how probability and statistics are used in decision making about immunisation and public health.

It is beneficial for learners to be able to use and apply school mathematics to real world situations and to be able to recognise the misinterpretation of data that they often meet in the media and in advertising. Everyone has been affected by the Covid19 pandemic and even very young children are aware of this. Life-changing decisions made by governments have, to different extents, been influenced by expert knowledge and understanding of probability and statistics.

DIAGNOSTIC ASSESSMENT FOR SECONDARY This should take about 5–10 minutes.
Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

A box contains 20 nails.
The table shows information about the length of each nail.

Length of nail (mm)	25	30	40	50	60
Number of nails	1	8	4	5	2

Jamila puts all 20 nails into a bag.
She takes at random one of the nails and records its length.
She replaces the nail in the bag.
She then takes at random a second nail from the bag and records its length.

Calculate the probability that the two nails she takes

each have a length of 60 mm

A: 4/400

B: 4/20

C: 2/20

D: 2/400

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Again ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. The concept is needed for the lesson to follow so explain the right answer.

The correct answer is: **A** The probability is $2/20 \times 2/20$
Possible misconceptions
B. It could be that the learner has added the probabilities rather than multiplying.
C. This is the probability of picking a 60mm nail each time.

Learning objectives

In doing this activity students will have an opportunity to review and consolidate knowledge and understanding of the use of Venn diagrams, and tree diagrams as aids to solving probability problems (where events are not necessarily independent).

Generic competences

In doing this activity students will have an opportunity to develop their understanding of probability and their problem solving skills in contexts that involve chance or risk.

Suggestions for teaching

Getting started: Copy the question and the contingency table on the board and ask the learners to work in pairs to fill in the figures (in percentages) in the contingency table corresponding to the information given about the problem and also to fill in the totals.

Then lead a class discussion in which, in response to your questions, learners suggest how the contingency table should be filled in. Make sure that all the learners copy the completed contingency table into their notebooks.

To encourage discovery and statistical / probabilistic talk: ask the learners to draw a Venn diagram to give the same information and to enter the information as fractions rather than percentages. Ask learners to notice which probabilities from the contingency table can be shown on the Venn Diagram and which are not explicitly shown but can be deduced. There is extra or redundant information in the contingency table which, for some learners. makes the table easier to read and to understand.

Lead a class discussion about filling in the Venn diagram. A very common source of confusion and misunderstanding is that some learners do not appreciate that the region outside the circles in the Venn diagram, within the rectangular frame, represents the complement of the sets shown inside the circles. Make sure that all the learners understand how the Venn diagram is constructed, what the different regions represent and how to find the proportion infected.

Then copy the partially filled in tree diagram onto the board and ask learners to copy it, to fill in the remainder of the information and to answer the remaining questions.

Finally ask questions and invite learners to come to the board, in pairs, and explain how they have completed the tree diagram and why, and how they have worked out the answers to all the questions. Finally summarise what has been learned in the lesson and make sure the learners understand that fractions, decimals and percentages are just alternative ways of representing and recording the same number.

Developing mathematical language: Taking time for discussion of the three diagrams will help to form a solid base on which to develop the more formal language of probability so that learners gain a good understanding of the probabilistic language of events, unions, intersections etc rather than learning the words in a rote manner. It empowers learners to realise that they use this sort of language in a range of domains and discussions in their everyday life. For example: You cannot get a driving license unless you are over 18 and you pass the driving test. Ask learners to think of more examples of this sort of language from their usual conversations.

Key questions

- If 20% of the population were inoculated, what percentage were not inoculated?
- What percentage of the whole population is inoculated and infected? What percentage is inoculated and not infected?
- What percentage of the whole population is not inoculated and infected? What percentage is not inoculated and not infected?
- Can you give that percentage as a fraction?
- Can you give that percentage as a decimal?

Follow up

The table gives the percentages of right and left handed males and females a random sample of people from a large population.

Sex \ Handedness	Right handed	Left handed	Total
Male	43	9	52
Female	44	4	48
Total	87	13	100

What is the probability that a female chosen at random from this population is left handed?

What is the probability that a male chosen at random from this population is left handed?

Carry out a survey of 100 people, either in your school or in your home area and record your data in a similar way. You might do this as a group project so that each person in your group just asks a few people and takes care to avoid the same person being asked more than once. Compare your data with the data given here.

Two Aces https://aiminghigh.aimssec.ac.za/years-9-12-two-aces/

If this then that https://aiminghigh.aimssec.ac.za/years-10-12-if-this-then-that/

Lucky Numbers https://aiminghigh.aimssec.ac.za/grades-7-to-12-lucky-numbers/

Mathsland Lottery

https://aiminghigh.aimssec.ac.za/grades-9-to-12-mathsland-lottery/