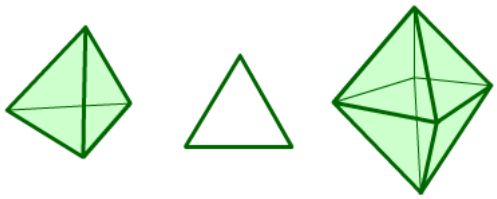
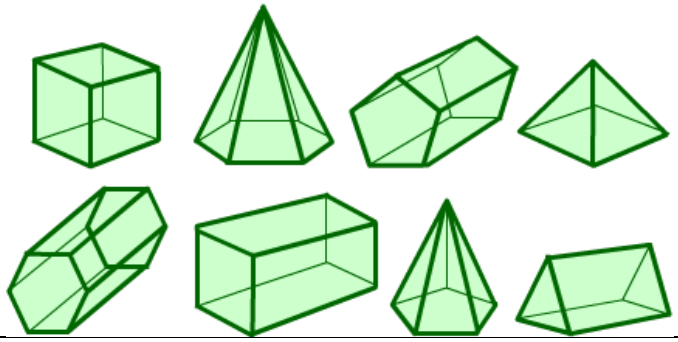
CHAIN OF POLYHEDRA

These 3-D objects, a tetrahedron and an octahedron, both have equilateral triangular faces.



Can you arrange these 8 polyhedra in a line so that every two polys next to each other have a face of the same shape. They do not need to be the same size.



HELP

It might be helpful make a list of the shapes of the faces in each of the 8 polyhedra using this set of polygons.

You could make a 2-way table:

Shape	Hexagonal face	Square face	Equilateral triangle face	Rectangular face	Pentagonal face	Isosceles triangle face
Cube						
Hexagonal based pyramid						
Pentagonal prism						

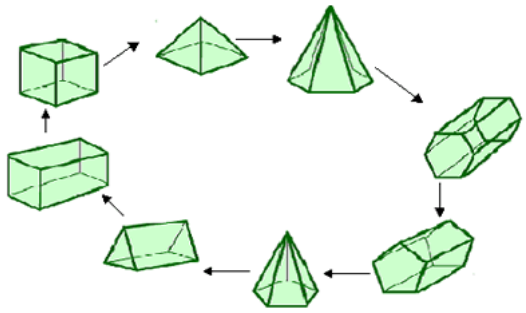
NEXT

(1) Is there more than one way to arrange the polyhedral in a ring so that the polys next to each other have at least one face that is the same shape?

(2) You could add some more 3D shapes and try to make a longer chain. Perhaps you could draw the shapes as in the problem.

NOTES FOR TEACHERS

SOLUTION



You can start anywhere in this ring. There are several other solutions. How many can your class find?

These tables are the work of school children and are re-produced from https://nrich.maths.org/2316/solution

	Square face	Hexagon face	Triangle face	Pentagon face	Rectangle face
Cube	**Yes**	No	No	No	No
Hexagon-based pyramid	No	**Yes**	**Yes**	No	No
Pentagonal prism	No	No	No	**Yes**	**Yes**
Square-based pyramid	**Yes**	No	**Yes**	No	No
Hexagonal prism	No	**Yes**	No	No	**Yes**
Cuboid	**Yes**	No	No	No	**Yes**
Pentagon-based pyramid	No	No	**Yes**	**Yes**	No
Trianglar prism	No	No	**Yes**	No	**Yes**

Other solution chains can be found using the tables

Shape	**Goes with**
Cube	Square-based pyramid, Cuboid
Hexagon-based pyramid	Hexagonal prism,Square-based pyramid,Pentagon-based pyramid,Trianglar prism
Pentagonal prism	Pentagon-based pyramid, Hexagonal prism, Cuboid, Trianglar prism
Square-based pyramid	Cube, Cuboid, Hexagon-based pyramid,Pentagon-based pyramid, Trianglar prism
Hexagonal prism	Hexagon-based pyramid, Pentagonal prism, Cuboid, Trianglar prism
Cuboid	Cube, Square-based pyramid,Pentagonal prism,Hexagonal prism,Trianglar prism
Pentagon-based pyramid	Pentagonal prism, Hexagon-based pyramid,Square-based pyramid,Trianglar prism
Trianglar prism	Hexagon-based pyramid ,Square-based pyramid,Pentagon-based pyramid,Pentagonal prism,Hexagonal prism,Cuboid

NRICH is a treasure trove for teachers because it provides thousands of learning activities together with children's solutions and suggestions for teaching and, in addition, many interactive games and helpful articles to read.

Why do this activity?

This is an excellent activity to start an inquiry based lesson and to engage learners in talking about and describing 3D shapes and relating them to the shapes of their 2D faces.

Learning objectives

In doing this activity students will have an opportunity to:

- learn the names and properties of some simple 3D objects;
- recognize 2D shapes and their properties;
- think mathematically to work out different possible solutions to the problem;
- develop visualisation and systematic working.

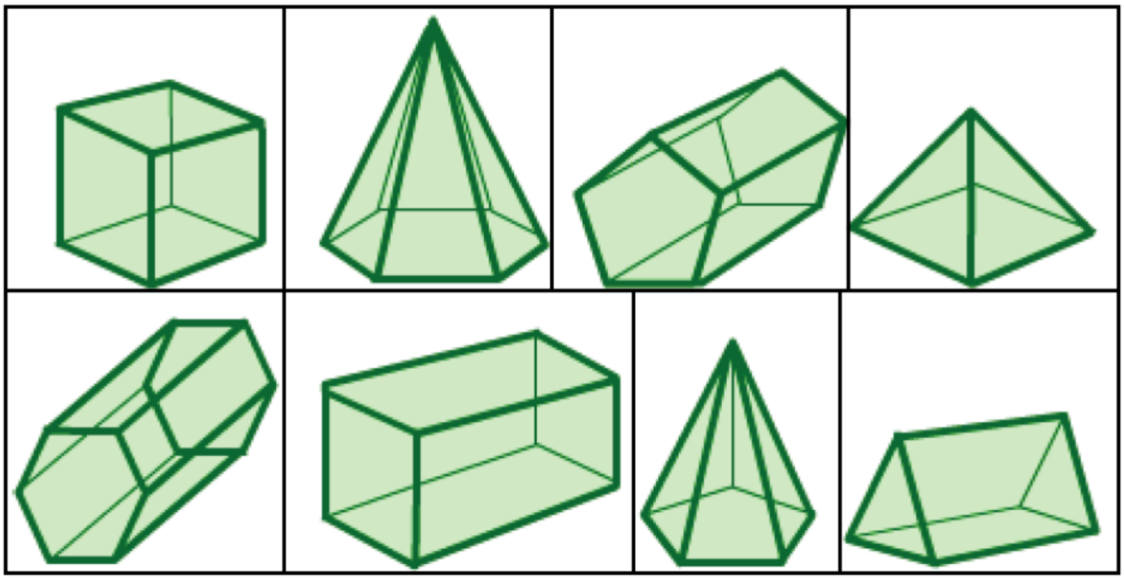
Generic competences

In doing this activity students will have an opportunity to:

- develop the skill of interpreting and creating visual images to represent concepts and situations;
- organise, analyse, and interpret information;
- analyze, reason and record ideas effectively.

Suggestions for teaching

Learners could work in pairs or groups, and use these cards to help them to arrange the polyhedra in a chain with matching faces.



After the learners have arranged the objects the teacher can ask individuals to explain why they have arranged them in the order that they have used.

Ask learners to say what they think is **the same** and what they think **is different** about the orders suggested.

Ask if the objects on the ends of the line can be matched in the same way. The learners will see that in every case this is so.

Ask them to think hard about how the objects could be arranged so that the objects on the end, that also match, are next to each other. Someone will probably suggest putting them in a circle or ring instead of a straight line. The teacher can demonstrate this then explain that this is called a **CYCLE** which is a very important pattern that often occurs in mathematics and in everyday life. Point out days of the week are a 7-cycle, the clock is a 24 hour cycle etc.

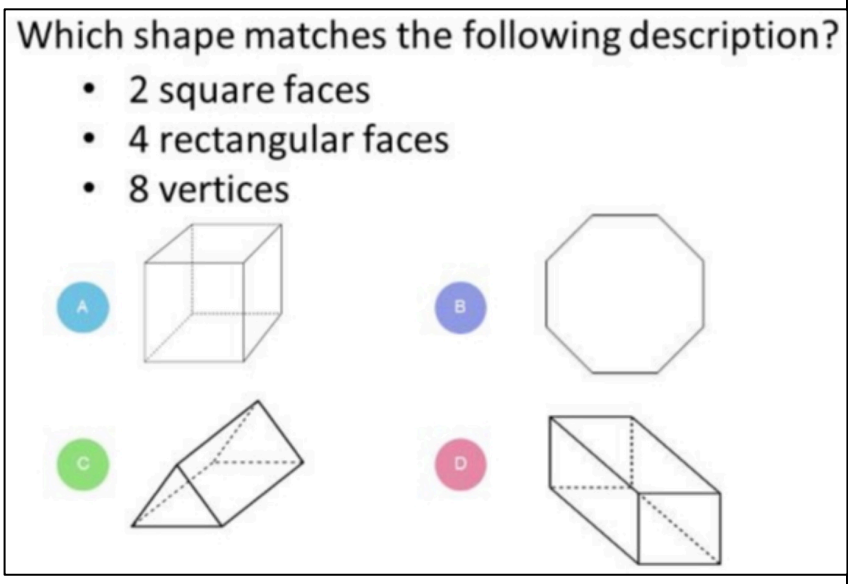
Key questions

- Would it help to list the shape of the faces of each 3D shape?
- Can you begin to link pairs of shapes together?
- Can you see another square/hexagon etc?
- If you have found a chain, can you find a ring or loop?

- If you have found a loop, how many different loops can you find?

Diagnostic Assessment This should take about 5–10 minutes at the start or end of a lesson. Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 for C and 4 for D".



1. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
4. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

D. is the correct answer.

Common Misconceptions

A. , **B.** or **C** Learners giving these answers appear not to understand what faces an vertices are in a solid.

https://diagnosticquestions.com

Follow up

See:

Paper Stick Tetrahedron

https://aiminghigh.aimssec.ac.za/years-4-8-paper-stick-tetrahedron/

Cube Nets https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/

Tets and Octs Puzzle

https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/

Icosahedron Puzzle https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/

Triangles to Tetrahedra

https://aiminghigh.aimssec.ac.za/years-8-11-triangles-to-tetrahedra/

Polyhedra By Paper Folding:

https://aiminghigh.aimssec.ac.za/years-6-to-12-polyhedra-by-paper-folding/

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