BEAUTIFUL NUMBERS

? × 9 + 7 = 88
? × 9 + 6 = 888
? × 9 + 5 = 8888
? × 9 + 4 = 88888
? × 9 + 3 = 888888
? × 9 + 2 = 8888888
? × 9 + 1 = 88888888
? × 9 + 0 = 888888888

Work out the numbers to replace the question marks.

How would you describe this pattern of calculations?

Can you explain how this pattern arises?

SOLUTION

9 x 9 + 7 = 88
98 x 9 + 6 = 888
987 x 9 + 5 = 8888
9876 x 9 + 4 = 88888
98765 x 9 + 3 = 888888
987654 x 9 + 2 = 8888888
9876543 x 9 + 1 = 88888888
98765432 x 9 + 0 = 888888888

If we look at the last two calculations we can see how the pattern arises. The other calculations are similar.

9876543 × 9 + 1 = 9876543 × (10-1) + 1

which can be written as

98 765 431
- 9 876 543
88 888 888

Similarly

98765432 × 9 + 1 = 98765432 × (10-1) + 0

can be written as

987 654 320
-98 765 432
888 888 888

NOTES FOR TEACHERS

Why do this activity?

This 'pre-algebra' activity leads learners into the method of inverse operations for solving equations without using any formal algebra. Learners will naturally start with the top line. Most learners will be able to find the first solution but many will use a trial and error method. Without any experience of algebra most learners will realise that, to find the other solutions, they have to use the inverse operations of subtraction and division, doing the same to both sides of the equation. The pattern in the answers is very pretty and it turns routine practice into more than usually interesting work.

Intended learning outcomes

- Development of understanding of the method for solving linear equations without the use of algebra.
- Appreciation of the concept of inverse operations and its application in simplifying numerical expressions.

Suggestions for teaching

You might start the lesson by writing **? × 9 + 6 = 888**
on the board and asking learners to find the correct number to replace the question mark. Starting with this rather than the top line reduces the chance that learners will simply guess the answer and use trial and error. Give the learners time to work on this and then to talk to a partner about it. Tell them you are going to ask them to explain their method.

Then have a class discussion about methods. Listen to what the learners have to say just making non-committal but encouraging remarks so that learners will not be deterred from making suggestions. Then respond so as to build on their ideas. When the learners seem to understand that by subtracting 6 and then dividing by 9 they get the answer 98 tell them that you are going to give them some more puzzles like this to solve in a similar way.

? × 9 + 7 = 88
? × 9 + 6 = 888
? × 9 + 5 = 8888
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? × 9 + 3 = 888888
? × 9 + 2 = 8888888
? × 9 + 1 = 88888888
? × 9 + 0 = 888888888

Either
(1) CLASS COOPERATION – LEARNERS SHARE THE WORK
To save time, to share the work around the class and to get the maximum impact from the pattern, write out the question pattern on the board and ask different groups of learners to complete lines 1and 3, and line4, line 5, line 6, line 7 and line 8.

You might assign the lines according to the ability of the learners so that the most able get line 8 and the weakest learners get lines 1 and 3. That way all the learners should be able to succeed and will finish their task at roughly the same time.

Then erase one question mark at a time and ask one of the learners to come to the board and fill in the number that should go in its place. Check that the other members of the group who worked out the number agree on the answer.

The learners will be surprised by the pattern that appears.

Finally have a class discussion about the results in which you ask the learners to describe what they notice.

Alternatively
(2) ALL LEARNERS DO MOST OR ALL THE CALCULATIONS
If you want everyone in the class to practice doing all the calculations write out the question pattern on the board and ask the learners to calculate the numbers that are needed to replace the question marks. Half the class might start at lines 1 and 3 and half start at line 8 and do the calculations in the reverse order. That way, when you check the answers, every line will have been done by some of the learners even if they have not finished them all.

Ask the learners to find the different numbers needed to replace all the question marks and to write their answers neatly in this pyramid shape.

Finally have a class discussion about the results in which you ask the learners to describe what they notice.

Key questions

What would you do first to simplify that?
How can you **undo** the + 6? (or whatever number is added on the line the learner is working on)
What could you do to both sides to get **? × 9 = some number?**
You have got **? ×9 = 882.** So how will you find out how many times 9 it should be?
You have got **? ×9 = 882.** So how will you undo that multiplication?
How can you check that answer?
What do you notice?

Possible extension

Replace the ? mark here:

987654321 × 9 + ? = 8888888888

Possible support

Add up the totals. What do you notice about the patterns in these calculations?

810	90×9	8100	900×9	81000	9000×9	810000	90000×9
+72	8×9	720	80×9	7200	800×9	72000	8000×9
?		+63	7×9	630	70×9	6300	700×9
		?		+54	6×9	540	60×9
				?		+45	5×9
						?	

8100000	900000×9	81000000	9000000×9	810000000	90000000×9
720000	80000×9	7200000	800000×9	72000000	8000000×9
63000	7000×9	630000	70000×9	6300000	700000×9
5400	600×9	54000	6000×9	540000	60000×9
450	50×9	4500	500×9	45000	5000×9
+36	4×9	360	40×9	3600	400×9
?		+27	3×9	270	30×9
		?		+18	2×9
				?	

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.				
	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6