



**AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES**  
**SCHOOLS ENRICHMENT CENTRE (AIMSSEC)**

**AIMING HIGH**

**BEAUTIFUL NUMBERS**

$$\begin{aligned} ? \times 9 + 7 &= 88 \\ ? \times 9 + 6 &= 888 \\ ? \times 9 + 5 &= 8888 \\ ? \times 9 + 4 &= 88888 \\ ? \times 9 + 3 &= 888888 \\ ? \times 9 + 2 &= 8888888 \\ ? \times 9 + 1 &= 88888888 \\ ? \times 9 + 0 &= 888888888 \end{aligned}$$

Work out the numbers to replace the question marks.

How would you describe this pattern of calculations?

Can you explain how this pattern arises?

**Help**

Add up the totals in the table below and write them in place of the question marks.

Notice that, you are given the equivalent multiple of 9 on each line to help you, for example  $810 = 90 \times 9$  and  $7200 = 800 \times 9$ .

What do you notice about the patterns in these calculations?

Now go back and try the puzzle again.

$\begin{array}{r} 810 \\ +72 \\ \hline ? \end{array}$	$\begin{array}{r} 90 \times 9 \\ 8 \times 9 \end{array}$	$\begin{array}{r} 8100 \\ 720 \\ +63 \\ \hline ? \end{array}$	$\begin{array}{r} 900 \times 9 \\ 80 \times 9 \\ 7 \times 9 \end{array}$	$\begin{array}{r} 81000 \\ 7200 \\ 630 \\ +54 \\ \hline ? \end{array}$	$\begin{array}{r} 9000 \times 9 \\ 800 \times 9 \\ 70 \times 9 \\ 6 \times 9 \end{array}$	$\begin{array}{r} 810000 \\ 72000 \\ 6300 \\ 540 \\ +45 \\ \hline ? \end{array}$	$\begin{array}{r} 90000 \times 9 \\ 8000 \times 9 \\ 700 \times 9 \\ 60 \times 9 \\ 5 \times 9 \end{array}$
$\begin{array}{r} 8100000 \\ 720000 \\ 63000 \\ 5400 \\ 450 \\ +36 \\ \hline ? \end{array}$	$\begin{array}{r} 900000 \times 9 \\ 80000 \times 9 \\ 7000 \times 9 \\ 600 \times 9 \\ 50 \times 9 \\ 4 \times 9 \end{array}$	$\begin{array}{r} 81000000 \\ 7200000 \\ 630000 \\ 54000 \\ 4500 \\ 360 \\ +27 \\ \hline ? \end{array}$	$\begin{array}{r} 9000000 \times 9 \\ 800000 \times 9 \\ 70000 \times 9 \\ 6000 \times 9 \\ 500 \times 9 \\ 40 \times 9 \\ 3 \times 9 \end{array}$	$\begin{array}{r} 810000000 \\ 72000000 \\ 6300000 \\ 540000 \\ 45000 \\ 3600 \\ 270 \\ +18 \\ \hline ? \end{array}$	$\begin{array}{r} 90000000 \times 9 \\ 8000000 \times 9 \\ 700000 \times 9 \\ 60000 \times 9 \\ 5000 \times 9 \\ 400 \times 9 \\ 30 \times 9 \\ 2 \times 9 \end{array}$		

**Extension**

Replace the ? mark here:

$$987654321 \times 9 + ? = 8888888888$$



## NOTES FOR TEACHERS

### SOLUTION

$$9 \times 9 + 7 = 88$$

$$98 \times 9 + 6 = 888$$

$$987 \times 9 + 5 = 8888$$

$$9876 \times 9 + 4 = 88888$$

$$98765 \times 9 + 3 = 888888$$

$$987654 \times 9 + 2 = 8888888$$

$$9876543 \times 9 + 1 = 88888888$$

$$98765432 \times 9 + 0 = 888888888$$

If we look at the last two calculations we can see how the pattern arises. The other calculations are similar.

$9876543 \times 9 + 1 = 9876543 \times (10-1) + 1$   
 which can be written as

$$\begin{array}{r} 98\ 765\ 431 \\ -\ 9\ 876\ 543 \\ \hline 88\ 888\ 888 \end{array}$$

Similarly

$98765432 \times 9 + 1 = 98765432 \times (10-1) + 0$   
 can be written as

$$\begin{array}{r} 987\ 654\ 320 \\ -98\ 765\ 432 \\ \hline 888\ 888\ 888 \end{array}$$

### Diagnostic Assessment

This should take about 5–10 minutes.

- Write the question on the board, say to the class:  
**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and **DO NOT** say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Which of the follow is equivalent to  $201 \times 9 - 9 \times 102$ ?

- A.  $99 \times 9$
- B.  $101 \times 9$
- C. 0
- D. 9

The correct answer is **A**.  
**Possible misconceptions:**

**B.** and **D** Students giving these answers could be guessing because they fail to understand commutativity and what the question is asking about.

**C.** Probably these students focussed on  $9 - 9$

<https://diagnosticquestions.com>



## Why do this activity?

This 'pre-algebra' activity leads learners into the method of inverse operations for solving equations without using any formal algebra. Learners will naturally start with the top line. Most learners will be able to find the first solution but many will use a trial and error method. Without any experience of algebra most learners will realise that, to find the other solutions, they have to use the inverse operations of subtraction and division, doing the same to both sides of the equation. The pattern in the answers is very pretty and it turns routine practice into more than usually interesting work.

## Learning objectives

In doing this activity students will have an opportunity to:

- development of understanding of the method for solving linear equations without the use of algebra;
- appreciation of the concept of inverse operations and its application in simplifying numerical expressions.

## Generic competences

*We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.*

In doing this activity students will have an opportunity to:

- **reason logically** and give explanations;
- **visualization** - develop the skill of noticing interpreting patterns and using them to solve problems.

## Suggestions for teaching

You might start the lesson by writing  $? \times 9 + 6 = 888$

on the board and asking learners to find the correct number to replace the question mark. Starting with this rather than the top line reduces the chance that learners will simply guess the answer and use trial and error. Give the learners time to work on this and then to talk to a partner about it. Tell them you are going to ask them to explain their method.

Then have a class discussion about methods. Listen to what the learners have to say just making non-committal but encouraging remarks so that learners will not be deterred from making suggestions. Then respond so as to build on their ideas. When the learners seem to understand that by subtracting 6 and then dividing by 9 they get the answer 98 tell them that you are going to give them some more puzzles like this to solve in a similar way.

*Either*

(1) CLASS COOPERATION – LEARNERS SHARE THE WORK

**To save time, to share the work around the class and to get the maximum impact from the pattern,** write out the question pattern on the board and ask different groups of learners to complete lines 1 and 3, and line 4, line 5, line 6, line 7 and line 8.

You might assign the lines according to the attainment levels of the learners so that the higher attaining get line 8 and the weakest learners get lines 1 and 3. That way all the learners should be able to succeed and will finish their task at roughly the same time.

Then erase one question mark at a time and ask one of the learners to come to the board and fill in the number that should go in its place. Check that the other members of the group who worked out the number agree on the answer.

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The learners will be surprised by the pattern that appears.

Finally have a class discussion about the results in which you ask the learners to describe what they notice.

*Alternatively*

(2) ALL LEARNERS DO MOST OR ALL THE CALCULATIONS

If you want everyone in the class to practice doing all the calculations write out the question pattern on the board and ask the learners to calculate the numbers that are needed to replace the question marks. Half the class might start at lines 1 and 3 and half start at line 8 and do the calculations in the reverse order. That way, when you check the answers, every line will have been done by some of the learners even if they have not finished them all.

Ask the learners to find the different numbers needed to replace all the question marks and to write their answers neatly in this pyramid shape.

Finally have a class discussion about the results in which you ask the learners to describe what they notice.

### Key questions

- What would you do first to simplify that?
- How can you **undo** the + 6? (or whatever number is added on the line the learner is working on)
- What could you do to both sides to get **? × 9 = some number?**
- You have got **? × 9 = 882**. So how will you find out how many times 9 it should be?
- You have got **? × 9 = 882**. So how will you undo that multiplication?
- How can you check that answer?
- What do you notice?

### Follow up

Check it <https://aiminghigh.aimssec.ac.za/years-4-to-7-checkit/>

**Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.**

For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

**Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.**

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6