AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

PROPERTIES OF 3D SHAPES is the theme in this

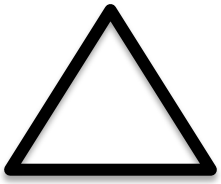
INCLUSION AND HOME LEARNING GUIDE

This Guide suggests related learning activities for all ages from 5 to 17+

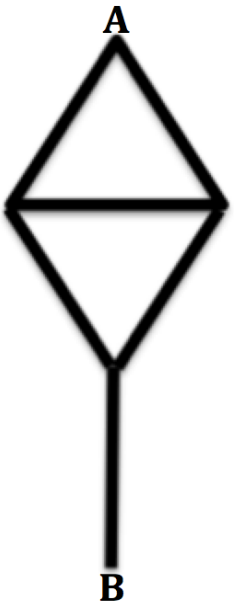
The PAPER STICK TETRAHEDRON activity was designed for Primary and Lower Secondary
Just choose whatever seems suitable for your group of learners.

PAPER STICK TETRAHEDRON

First watch this video:
https://www.youtube.com/watch?v=UdG-L9bplVo&t=6s



Make 6 paper sticks, all the same length, by tightly rolling pieces of paper around a piece of string and securing it with sticky tape. See the video: https://www.youtube.com/watch?v=iaJ6EitIGKU&t=31s
Stick the string across the paper before you start rolling to stop it coming out once the stick has been rolled up.



Take 3 sticks and make a triangle by tying the string together.

The edges of the triangle are the same length. What about the angles?

Now tie the other 3 sticks to your triangle and make this shape.
What do you notice about this shape?

Can you 4 triangles from 2 triangles? Try it.

Tie the end B to the point A.

What shape do you get now? Describe your new shape.

HELP

Work with a partner so that you can help each other.

NEXT

Make an octahedron using 12 sticks. Count the faces, edges and vertices.
What do you notice about the octahedron?
See TETS AND OCTS PUZZLE
https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/

Resources: scrap paper, string, sticky tape, scissors for making paper sticks.

INCLUSION AND HOME LEARNING GUIDE
THEME: Properties of 3D shapes

Young children

Make the tetrahedron with the young children and talk about it. You might like to paint and decorate it and perhaps hang it up.

Upper Primary and Lower Secondary

First watch the video together https://www.youtube.com/watch?v=UdG-L9bplVo&t=6s. Then give each learner 3 sheets of ordinary scrap paper and 3 pieces of string. If your standard paper size is A4 then cut it in half to make A5 size; this will be ideal for the paper sticks. Show the children how to make paper sticks, all the same length, by tightly rolling a piece of paper around a piece of string and securing it with selotape. Start by sticking the string down before you start rolling the paper to stop the string being pulled out of the stick once it is rolled up. If you have not made paper sticks before then watch the video : https://www.youtube.com/watch?v=iaJ6EitIGKU&t=31s.

The sticks made from an A5 sheet of paper will be 148 mm long. You can use other paper and make sticks of any length you choose.

If you have a group the learners could work in pairs and between them start by making a triangle using 3 sticks and tying the string together tightly at the vertices. Then talk about what the children notice about the triangle. This is a chance for them to learn that the edges are all equal in length and so the triangle is called EQUILATERAL (*equi* meaning equal and *lateral* referring to the edges). They should notice that the angles are also equal and could you talk about the angles being 60^{o}. You may want to introduce the word REGULAR.

Then give the learners the instructions on page 1, or give written instructions. Ask the learners to follow the instructions and make a tetrahedron. Then they should decide, with their partner, how they would describe the shape they have made and make notes about what they notice about it.

Then talk about what they notice about the tetrahedron. You can decide to discuss ALL, or only SOME of, the properties given in the solution on page 1. You might decide to select certain properties and delay talking about other properties (for example the symmetries) according to the age of your class and what they already know. You could make a list as learners suggest the various properties. You should keep asking: "What *else* do you notice?"

Key questions

- What do you notice about that shape?
- What can you say about the lengths of the edges?
- What can you say about the angles?

- Is that shape 2 dimensional or 3 dimensional?
- How many faces does it have? How many vertices? How many edges?
- Are all the angles equal? Are all the edges equal in length? If your answer to both questions is YES then this is a REGULAR polyhedron (poly/many , hedron/faces).
- Why do you think this shape is called a tetrahedron (tetra/four)?

Years 10 - 13

These students should be able to describe at least some of the symmetries of the tetrahedron. This may not occur in their examinations but symmetry is a hugely important topic in mathematics and its applications, for example in physics and in chrystallography.

Some Year 13 students may learn about groups and the symmetries of a tetrahedron which include 12 rotations and 12 reflections making a group with 24 elements called S_4. This group is isomorphic to the group of permutations of 4 objects {1, 2, 3, 4}. The rotations form the subgroup A_4 made up of permutations of 4 objects having an even number of transpositions. The reflections form a coset of S_4.
See https://www.youtube.com/watch?v=2g7gYebaIBs

More Key Questions

SYMMETRIES

- Does the tetrahedron have any symmetry? What sort of symmetry? Can you describe the symmetry?
- How could we cut through the middle of the tetrahedron to cut it into 2 identical halves that are mirror images of each other?
- Can you tell me how to turn the tetrahedron around so this corner (vertex) goes to that corner?
- If we rotate the tetrahedron 3 times (by 120^o) to show rotational symmetry of order 3 what line stays fixed?
- If we rotate the tetrahedron 2 times (by 180^o) to show rotational symmetry of order 2 what line stays fixed?

SOLUTION

The 6 paper sticks make a tetrahedron.

Learners should notice:

- All the edges are the same length.
- It has 4 triangular faces.
- The faces are equilateral triangles.
- It has 6 edges.
- It has 4 vertices.
- All the angles are equal and all the lengths are equal so the shape is REGULAR.
- The shape is 3 dimensional.

All the above is in most school mathematics curricula.
What follows is in some curricula, often as an option, for Year 13.

Imagine the tetrahedron with one face on a table.
After **A SYMMETRY TRANSFORMATION,** the tetrahedron returns to its original position occupying the **same space**.

The regular tetrahedron has ROTATION SYMMETRY with 12 ROTATIONS:

- 8 rotations of order 3, by 120^o and 240^o about the 4 lines through each vertex and the centre of the opposite face.
- 3 rotations of order 2, by180^o about the 3 lines joining the midpoints of opposite edges.
- 1 identity rotation.

The regular tetrahedron has REFLECTION SYMMETRY with 12 REFLECTIONS:

- 6 reflections in 6 mirror planes, each plane containing an edge of the tetrahedron and passing through the midpoint of the opposite edge.
- 6 rotational reflections, each a combination of a 90^o or a 270^o rotation about an axis joining the midpoints of opposite edges, combined with a reflection in a plane half-way between the two midpoints.

Why do this activity?

This activity enables learners to make models of a regular tetrahedron that they can handle and examine so that they see it from every direction. It gives learners an opportunity to notice the properties for themselves which should then be discussed in class, listed and remembered.

Learning objectives

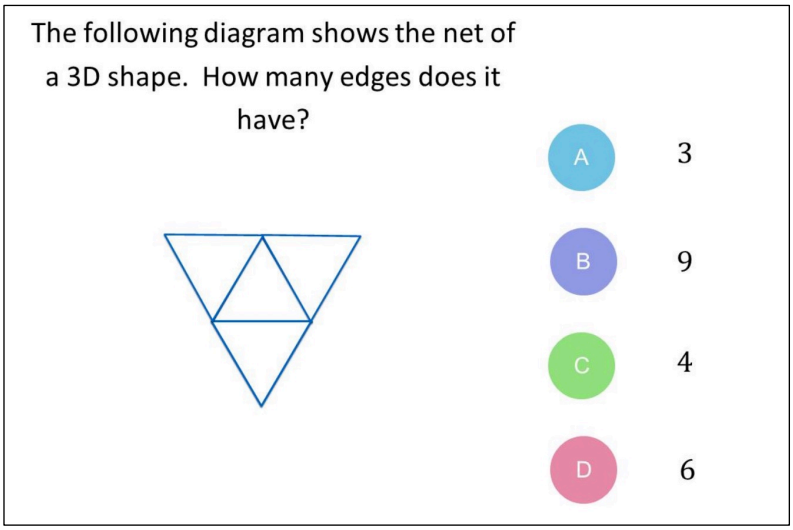
In doing this activity students will have an opportunity to:

- experience making a model of a tetrahedron.
- develop visualization and the ability to describe characteristics and properties of a tetrahedron including symmetries.

Generic competences

In doing this activity students will have an opportunity to develop skills of observation.

DIAGNOSTIC ASSESSMENT Use this quiz after the lesson.
Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and get a better understanding.
3. If you have a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Again ask the learners to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change, and who gave right and wrong answers

D. is the correct answer.
Common Misconceptions
A. Counting only the 3 edges of the outer boundary of the net
B. Counting as 2 edges those edges that will be joined together
C. Counting vertices rather than edges. https://diagnosticquestions.com

Follow up

TETS AND OCTS PUZZLE
https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/
CUT NETS https://aiminghigh.aimssec.ac.za/years-6-10-cut-nets/
TET TROUBLE https://aiminghigh.aimssec.ac.za/years-9-10-tet-trouble/
TETRASQUARE https://aiminghigh.aimssec.ac.za/years-11-12-tetrasquare/