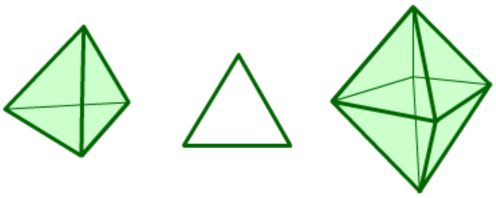
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

**The Chain of Polyhedra Inclusion and Home Learning Guide** is part of a Learning Pack downloadable from the AIMING HIGH website https://aiminghigh.aimssec.ac.za
It provides related activities for inclusion in school lessons, and guidance for home learning, for all ages and learning stages from pre-school to school-leaving, on the Common Theme **Matching Corresponding Properties**
Suggestions for school lessons are given in the separate Notes for Teachers.

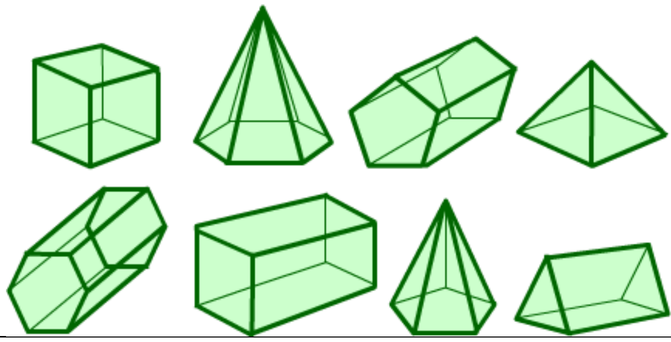
Choose what seems suitable for the age or attainment level of your learners.

## CHAIN OF POLYHEDRA

These 3-D objects, a tetrahedron and an octahedron, both have equilateral triangular faces.



Can you arrange these 8 polyhedra in a line so that every two polys next to each other have a face of the same shape. They do not need to be the same size.



## HELP

It might be helpful make a list of the shapes of the faces in each of the 8 polyhedra using this set of polygons.

You could make a 2-way table:

| Shape | Hexagonal face | Square face | Equilateral triangle face | Rectangular face | Pentagonal face | Isosceles triangle face |
|---|---|---|---|---|---|---|
| Cube | | | | | | |
| Hexagonal based pyramid | | | | | | |
| Pentagonal prism | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

## NEXT

(1) Is there more than one way to arrange the polyhedra in a ring so that the polys next to each other have at least one face that is the same shape?

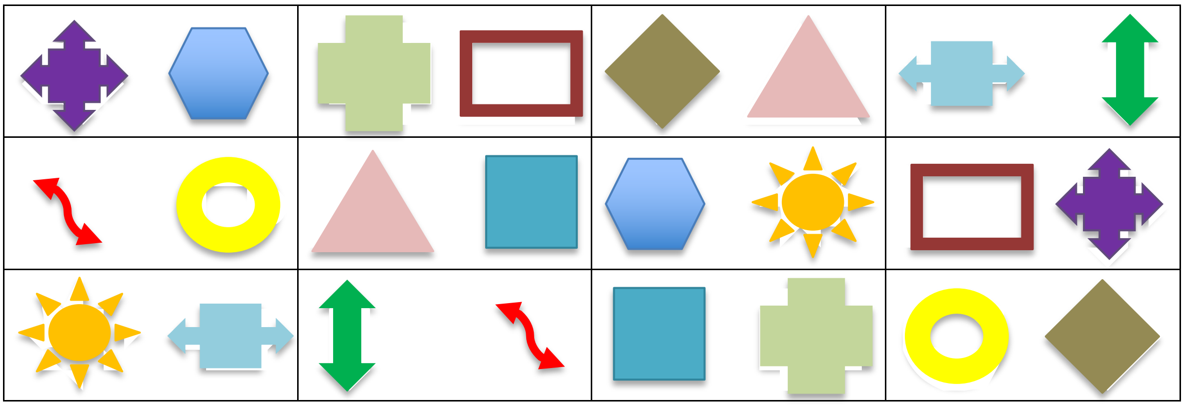
(2) You could add some more 3D shapes and try to make a longer chain. Perhaps you could draw the shapes as in the problem.

# INCLUSION AND HOME LEARNING GUIDE

# THEME: MATCHING CORRESPONDING PROPERTIES

## Early Years

Cut out the 12 cards.



Talk about the shapes.

Ask the children to describe the shapes. What do they notice about them?

Look at pairs of shapes that match in some respects and not in other ways. What is the same and what is different about them?

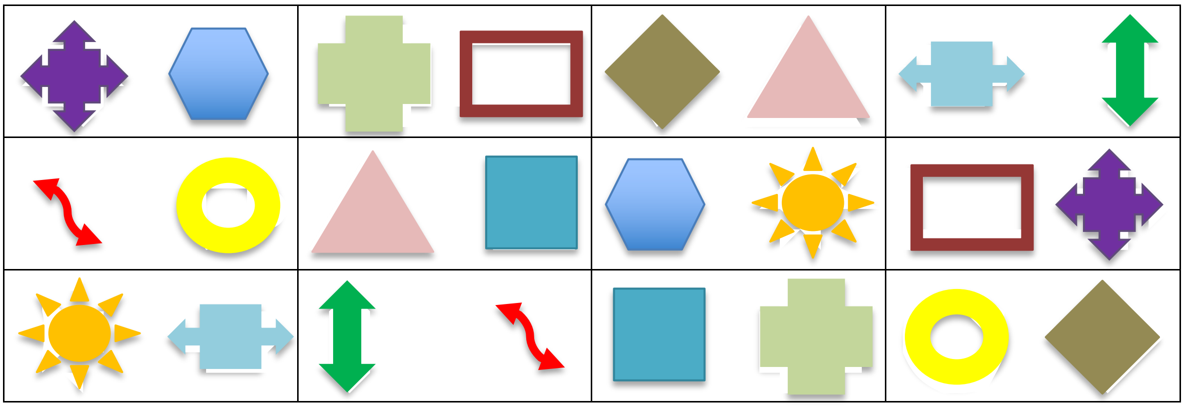
Ask the children to match the cards, placing them end to end in a line, so that the shapes on cards placed next to each other correspond.

Then ask them to arrange their chain of cards into a loop with no start and no end point.

Is there more than one way to do this?

## Lower Primary

Cut out the 12 cards.



Talk about the shapes.

Ask the children to describe the shapes. What do they notice about them?

Look at pairs of shapes that match in some respects and not in other ways. What is the same and what is different about them?

Ask the children to match the cards, placing them end to end in a line, so that the shapes on cards placed next to each other correspond.

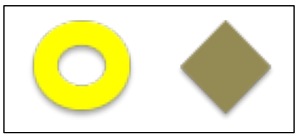
Then ask them to arrange their chain into a loop with no start and no end point.

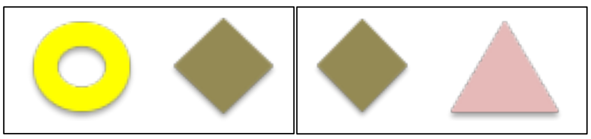
Is there more than one way to do this?

Arrange the cards in line in as many different orders as possible noting that two orders are the same if they form the same loop.

### **Multi-match Shape Game** for 2, 3 or 4 players

Players take it in turns to place a card in line end to end so that the shapes correspond and they score the number of edges of the shapes on the domino.
You can use any single criteria for matching the dominoes (for example the number of edges, the actual shape, whether the shape includes arrows, having a hole in the middle etc.) but the player has to explain exactly how the shapes match.

For example the domino  scores 6.

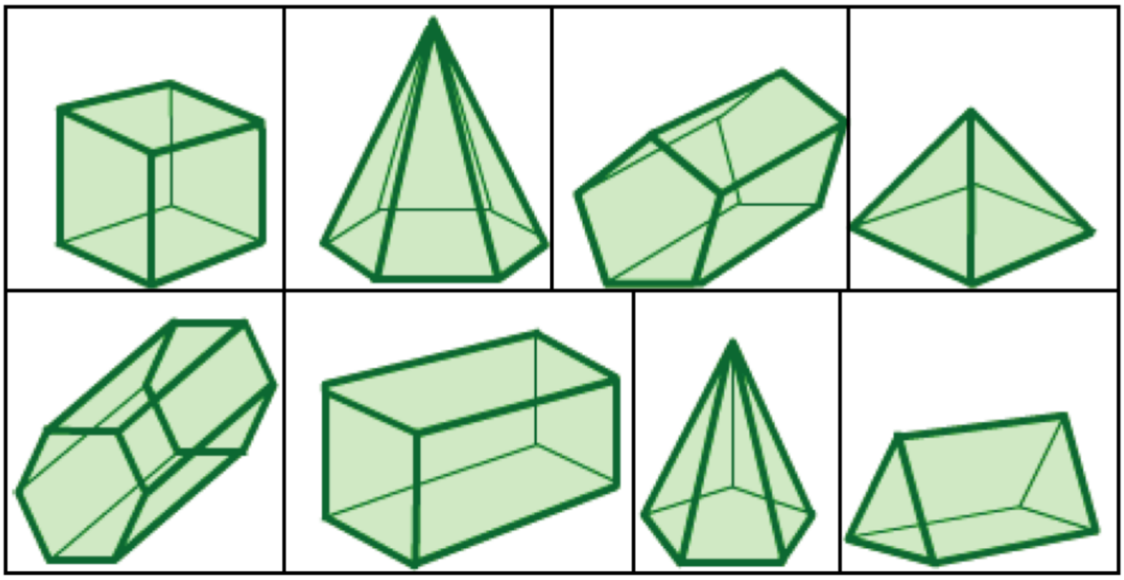


These two dominoes match because the squares are the same. The domino with the square and triangle scores 7.

## Upper Primary and Lower Secondary

As a warm-up, and to find out what learners know about 3D shapes, start with the Diagnostic Quiz.

Then ask the learners to arrange these 8 polyhedra in a line so that every two polys next to each other have a face of the same shape. They do not need to be the same size. Learners could work in pairs or groups. You might like to give each group a copy of these cards with scissors to cut them out. Re-arranging the cards makes it easier to match faces to form the required chain.



After the learners have arranged the objects you can ask individuals to explain why they have arranged them in the order that they have used.

Ask learners to say what they think is **the same** and what they think **is different** about the orders suggested.

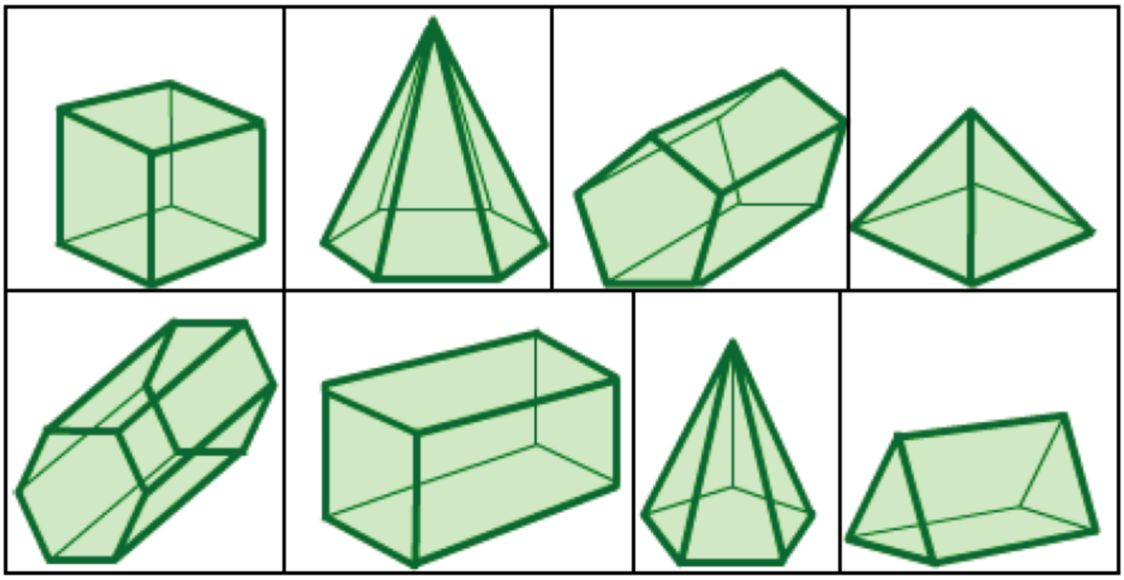
Ask if the objects on the ends of the line can be matched in the same way. The learners will see that in every case this is so.

Ask them to think hard about how the objects could be arranged so that the objects on the end, that also match, are next to each other. Someone will probably suggest putting them in a circle or ring instead of a straight line.

The teacher can demonstrate this then explain that the loop is called a **CYCLE** which is a very important pattern in mathematics. Cycles often occurs in mathematics and in everyday life, for example: days of the week are a 7-cycle, the clock is a 24 hour cycle etc.

## Upper Secondary

(1) Cut out these cards and arrange them in a line so that so that every two polyhedra next to each other have a face of the same shape. They do not need to be the same size. Observe that the polyhedra on the ends of your line also match. Form your line into a closed loop. This is called a **CYCLE** which is a very important pattern that often occurs in mathematics and in everyday life. For example: days of the week are a 7-cycle, the clock is a 24 hour cycle etc.



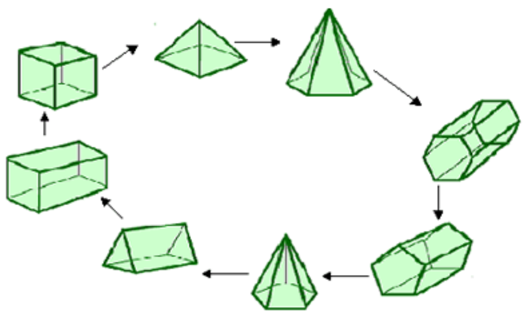
Complete the table given in the HELP Section on Page 2. Also make a table in which you list the 8 polyhedra and, for each one, give a list of matching polyhedra.

(2) Describe and explain some cyclic patterns that occur in mathematics, for example in the last digits of multiples of a given number, or in fractions as, in the decimal expansion of $\frac{2}{7}$.

(3) Investigate Happy Numbers. Start with any 2 digit number, square the digits and add the squares. A happy number is a number which eventually reaches 1 when replaced by the sum of the squares of each digit. For instance, 13 is a happy number because

$1^2 + 3^2 = 10$ and $1^2 + 0^2 = 1$. On the other hand, 4 is not a happy number because the sequence starting with $4^2 = 16$ and $1^2 + 6^2 = 37$ never reaches 1. Instead the sequence comes back to the number that it started from, and so the process continues in an infinite cycle without ever reaching 1.

## SOLUTION



You can start anywhere in this ring. There are several other solutions. How many can your class find?

These tables are the work of school children and are re-produced from [https://nrich.maths.org/2316/solution](https://nrich.maths.org/2316/solution)

| | Square face | Hexagon face | Triangle face | Pentagon face | Rectangle face |
|---|---|---|---|---|---|
| Cube | **Yes** | No | No | No | No |
| Hexagon-based pyramid | No | **Yes** | **Yes** | No | No |
| Pentagonal prism | No | No | No | **Yes** | **Yes** |
| Square-based pyramid | **Yes** | No | **Yes** | No | No |
| Hexagonal prism | No | **Yes** | No | No | **Yes** |
| Cuboid | **Yes** | No | No | No | **Yes** |
| Pentagon-based pyramid | No | No | **Yes** | **Yes** | No |
| Trianglar prism | No | No | **Yes** | No | **Yes** |

Other solution chains can be found using the tables

| **Shape** | **Goes with** |
|---|---|
| Cube | Square-based pyramid, Cuboid |
| Hexagon-based pyramid | Hexagonal prism,Square-based pyramid,Pentagon-based pyramid,Trianglar prism |
| Pentagonal prism | Pentagon-based pyramid, Hexagonal prism, Cuboid, Trianglar prism |
| Square-based pyramid | Cube, Cuboid, Hexagon-based pyramid,Pentagon-based pyramid, Trianglar prism |
| Hexagonal prism | Hexagon-based pyramid, Pentagonal prism, Cuboid, Trianglar prism |
| Cuboid | Cube, Square-based pyramid,Pentagonal prism,Hexagonal prism,Trianglar prism |
| Pentagon-based pyramid | Pentagonal prism, Hexagon-based pyramid,Square-based pyramid,Trianglar prism |
| Trianglar prism | Hexagon-based pyramid ,Square-based pyramid,Pentagon-based pyramid,Pentagonal prism,Hexagonal prism,Cuboid |

NRICH is a treasure trove for teachers because it provides thousands of learning activities together with children's solutions and suggestions for teaching and many interactive games and helpful articles to read.

## Key Questions

- Would it help to list the shape of the faces of each 3D shape?
- Can you begin to link pairs of shapes together?
- Can you see another square/hexagon etc?
- If you have found a chain, can you find a ring or loop?
- If you have found a loop, how many different loops can you find?

## Why do this activity?

This is an excellent activity to start an inquiry based lesson and to engage learners in talking about and describing 3D shapes and relating them to the shapes of their 2D faces.

## Learning objectives

In doing this activity students will have an opportunity to:

- learn the names and properties of some simple 3D objects;
- recognize  2D shapes and their properties;
- think mathematically to work out different possible solutions to the problem;
- develop visualisation and systematic working.

## Generic competences

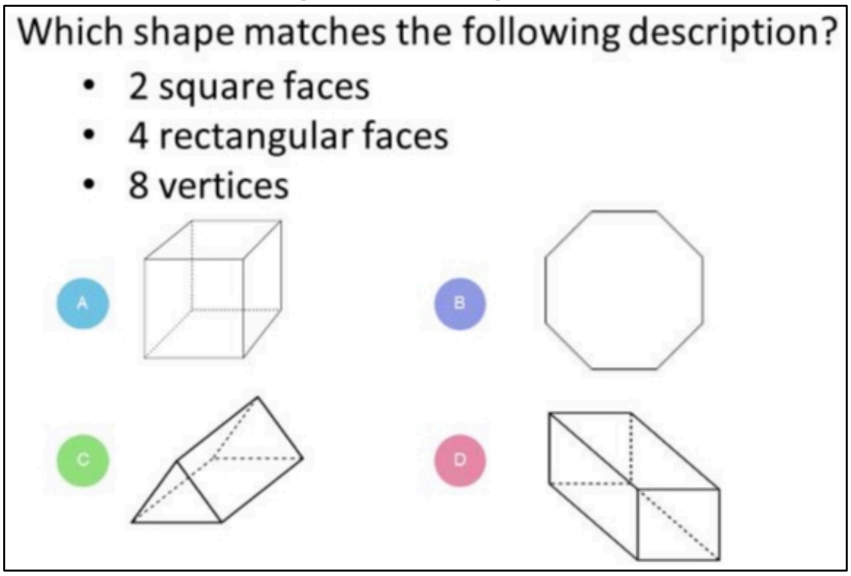
In doing this activity students will have an opportunity to:

- develop the skill of interpreting and creating visual images to represent concepts and situations;
- organise, analyse, and interpret information;
- analyze, reason and record ideas effectively.

**DIAGNOSTIC ASSESSMENT** This can be done before or after the lesson and as a group as described below, or the question can be answered individually.

Show this question and say:

**"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".**




1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2. It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain a better understanding.
3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: **D**

**A.** , **B.** or **C** Learners giving these answers appear not to understand what faces an vertices are in a solid.

https://diagnosticquestions.com

## Follow up

See:

Paper Stick Tetrahedron

https://aiminghigh.aimssec.ac.za/years-4-8-paper-stick-tetrahedron/

Cube Nets
https://aiminghigh.aimssec.ac.za/years-6-10-cube-nets/
Tets and Octs Puzzle
https://aiminghigh.aimssec.ac.za/years-7-10-tets-and-octs-puzzles/
Icosahedron Puzzle https://aiminghigh.aimssec.ac.za/years-7-10-icosahedron-puzzle/
Triangles to Tetrahedra
https://aiminghigh.aimssec.ac.za/years-8-11-triangles-to-tetrahedra/
Polyhedra By Paper Folding:
https://aiminghigh.aimssec.ac.za/years-6-to-12-polyhedra-by-paper-folding/
This problem is adapted from the NRICH task Chain of Polyhedra with permission of the University of Cambridge. 



Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

| | Lower Primary Approx. Age 5 to 8 | Upper Primary Age 8 to 11 | Lower Secondary Age 11 to 15 | Upper Secondary Age 15+ |
|---|---|---|---|---|
| South Africa | Grades R and 1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| East Africa | Nursery and Primary 1 to 3 | Primary 4 to 6 | Secondary 1 to 3 | Secondary 4 to 6 |
| USA | Kindergarten and G1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| UK | Reception and Years 1 to 3 | Years 4 to 6 | Years 7 to 9 | Years 10 to 13 |