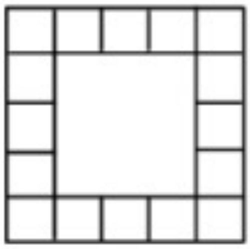
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

SQUARES PATTERN

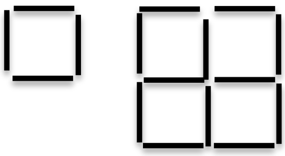
This square of squares pattern has edge length 5 squares. You could make this pattern with 48 matchsticks.



Work out the number of edge squares and the number of lines (matchsticks) needed to make the pattern with:

- side length 6
- side length 25
- side length 100
- side length n

SOLUTION



As with many problem solving activities it may help to start with the special cases n = 1, 2, 3 etc. Here 1 and 2 may be exceptional in that they do not have a hole inside so we need to be a bit cautious.

There are other ways to visualise how these patterns are made up and each method leads to the same sequence but may arrive at a different version of the formula. Allowing learners to develop their own methods provides a good learning experience. This can be used to motivate the algebraic work necessary to show that the different versions of the formula are equivalent.

We can think of making the squares by putting together C shapes facing different directions – try it!

To work out the number of squares you can subtract the number of inner squares missing inside the pattern from the number that would be in the block of squares, so for edge length 5, this is 25 – 9 = 16 and for edge length 6 this is 36 – 16 = 20.

The sequence of the number of squares from n = 1 to 6 is:
1, 4, 8, 12, 16, 20 ,…

And, for the number of line segments the sequence is:
4, 12, 24, 36, 48, 60,…

For the pattern of side length 25 there are
$25^2 - 23^2 = (25 - 23)(25 + 23) = 2 \times 48 = 96$ squares
made by $3 \times 96 = 288$ short line segments.

For the pattern of side length 100 there are
$100^2 - 98^2 = (100 - 98)(100 + 98) = 2 \times 198 = 396$ squares
made by $3 \times 396 = 1188$ short line segments.

For the pattern of side length n there are

$$\begin{aligned} n^2 - (n-2)^2 &= (n - (n-2))(n + (n - 2)) \\ &= 2 \times (2n - 2) \\ &= 4n - 4 \\ &= \mathbf{4(n-1)} \textbf{ squares} \end{aligned}$$

made by $3 \times (4n - 4) =$ **12(n – 1) short line segments**.

NOTES FOR TEACHERS

Why do this activity?

This problem looks at patterns and challenges learners to describe them clearly - verbally, numerically and algebraically. It does not assume prior knowledge of algebra and could be a good way to introduce, practise or assess algebraic fluency. It also provides a good introduction or review of the difference of two squares formula.

Similar-looking questions are often asked, expecting an approach that uses number sequences for finding a formula for the *nth* term. This problem deliberately bypasses all that, instead focusing on the structure of the pattern so that the algebraic expressions emerge naturally from that structure.

Possible approach

Draw the square of squares on the board by drawing short lines as if making the pattern from matchsticks or toothpicks. Then say: "I have drawn a matchstick square of squares on the board, and I would like you to make a rough copy of it - no need to use a ruler."
Once everyone has sketched the image - "Can anyone describe the order in which they drew the lines?"
"Without counting individual matches can you say how many matchsticks there are in your drawing?"

Collect at least three different methods, selecting learners who you know have something new to offer. For each method, draw it on the board (perhaps using colours to emphasise the order in which it was drawn) and pose the following questions:
"How would pattern with 6 squares on the edge be drawn using this method?"
“How many little squares would there be in the pattern?”
"How many matchsticks would be needed altogether?"
"What if there were 25 squares?"
"Or a hundred squares?"
"Or a million squares?"
"Or *n* squares?"

The answers to these questions could be recorded on the board, so that the results and the algebraic expressions emerging from each method can be compared at the end.

Some similar patterns are given below in the “**Possible extension’** section that you might use for a follow-on where the learners work in small groups.

Key questions

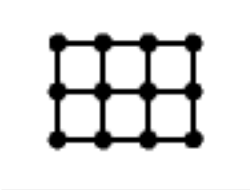
Can you see a pattern in the image?
How might you draw it?
How could you tell another person how to draw it over the phone when they can’t see you or the pattern?
Can you tell how the person drew the pattern from the way they write the calculation?
How does your formula relate to the structure of the original problem?

Possible extension

This problem lends itself to collaborative working, both for learners who are inexperienced at working in a group and learners who are used to working in this way.

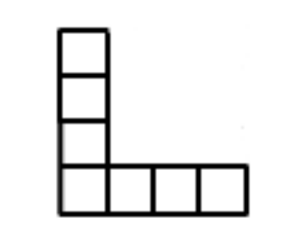
Here are some patterns that can be the basis of similar work. Each time the challenge is to extend the pattern from 5 to 6 and then to 25 and then 100 and then to n and to find the sequences and formulas that arise. Select some of these tasks and hand them out. You might want all groups to work on the same task(s), or you may want different groups to attempt different tasks. There are four different tasks and you may want to start with the Seven Squares task given in the **Possible support** section.

RECTANGLE
This rectangle has height 2 and width 3.



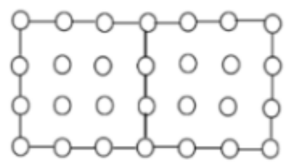
Work out the perimeter, the number of dots, and the number of lines needed to draw a rectangle with:
* height 2 and width 6
* height 2 and width 25
* height 2 and width 100
* height 2 and width n

L SHAPE
This L shape has height 4 and width 4.



Work out the perimeter, the number of squares, and the number of lines needed to draw an L shape with:
* height 6 and width 6
* height 25 and width 25
* height 100 and width 100
* height n and width n

TWO SQUARE PATTERN
This pattern is made from two joined squares with side length 3.



Work out the number of lines and the number of dots needed to draw the pattern of joined squares with:
* side length 6
* side length 25
* side length 100
* side length n

Explain that by the end of the session each group will be expected to report back to the rest of the class, showing how they saw the patterns growing, and how this helped them to work out the hundredth pattern and how they arrived at an algebraic expression. Exploring the full potential of these tasks is likely to take more than one lesson, allowing time in each lesson for students to feed back ideas and share their thoughts and questions.

Make sure that while groups are working they are reminded of the need to be ready to present their findings at the end, and that all are aware of how long they have left.

Ideally each group will record their diagrams, reasoning and generalisations on a large flipchart sheet in preparation for reporting back. Subsequently these can be put up on the classroom wall. There are many ways that groups can report back. Here are just a few suggestions:

- Every group is given a couple of minutes to report back to the whole class. Students can seek clarification and ask questions. After each presentation, students are invited to offer positive feedback. Finally, students can suggest how the group could have improved their work on the task.
- Everyone's posters are put on display at the front of the room, but only a couple of groups are selected to report back to the whole class. Feedback and suggestions can be given in the same way as above. Additionally, students from the groups which don't present can be invited to share at the end anything they did differently.
- Two people from each group move to join an adjacent group. The two "hosts" explain their findings to the two "visitors". The "visitors" act as critical friends, requiring clear mathematical explanations and justifications. The "visitors" then comment on anything they did differently in their own group.

Possible support

By working in groups with clearly assigned roles we are encouraging learners to take responsibility for ensuring that everyone understands before the group moves on.

This is a good pattern to start with:

SEVEN SQUARES
This pattern is made from matchsticks.
How many squares are there?
How many matchsticks are needed?
How would you draw it?

What about a line of 8 squares?
What about a line of 25 squares?
What about a line of 100 squares?
What about a line of n squares?

A teacher's comments after using this activity:
"It gave rise to much discussion about how to describe the patterns. It led naturally to building algebraic expressions and seeing them as easily understandable ways to record the patterns. It provided motivation for checking that the different algebraic expressions (used to describe the different ways in which a pattern can be built) are in fact equivalent."
"Some students succeeded in building the patterns and working numerically, but were not yet ready to work algebraically, while other students progressed to finding, and even simplifying, formulae for the patterns. All students experienced success and there was appropriate challenge in this problem for everyone."