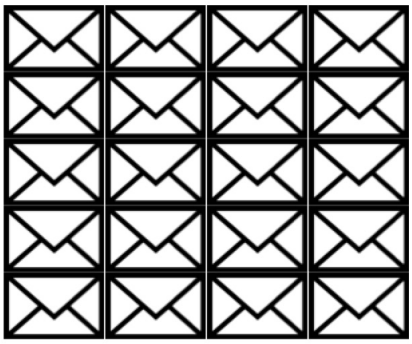
IF THIS THEN THAT



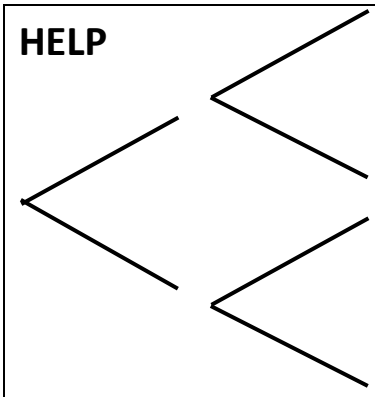
In front of you there are 20 envelopes.

Eight of the envelopes each contain 5 blue and 3 red sheets of paper.

The other 12 envelopes each contain 6 blue and 2 red sheets of paper.

You choose one envelope at random. Then you choose a sheet of paper from it at random. What is the probability that this sheet of paper is red?

HELP



Draw a tree diagram and fill in all the information given.

What is the probability that you will choose an envelope with 5 red and 3 blue?

What is the probability that you will choose an envelope with 2 red and 6 blue?

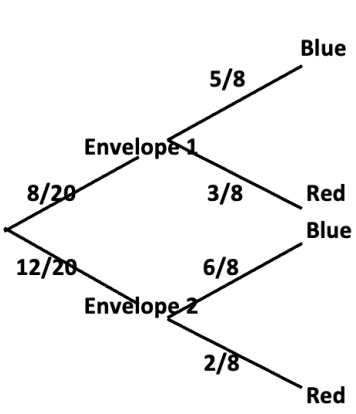
If you have chosen one sort of envelope rather than the other sort what difference does it make to the probability of choosing a sheet of red paper from it?

NEXT

Make up some extensions of this question of your own and find the answers. Then exchange your questions with a partner. Can you solve each other's questions? For example, if you choose an envelope at random and take out 2 sheets of paper, what is the probability that they are both red?

NOTES FOR TEACHERS

SOLUTION



PROBABILITY

P(Envelope 1 and Blue) = 8/20 × 5/8 = 1/4

P(Envelope 1 and Red) = 8/20 × 3/8 = 3/20

P(Envelope 2 and Blue) = 12/20 × 6/8 = 9/20

P(Envelope 2 and Red) = 12/20 × 2/8 = 3/20

Draw a tree diagram and fill in the information.

The prob ability that the sheet chosen is red = 3/20 + 3/20 = 3/10

Why do this activity?

This is a simple probability problem that gives learners practice in using tree diagrams. It could be introduced to review the idea of tree diagrams.

Learning objectives

- Practice in solving probability problems.
- Informal introduction to conditional probability.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

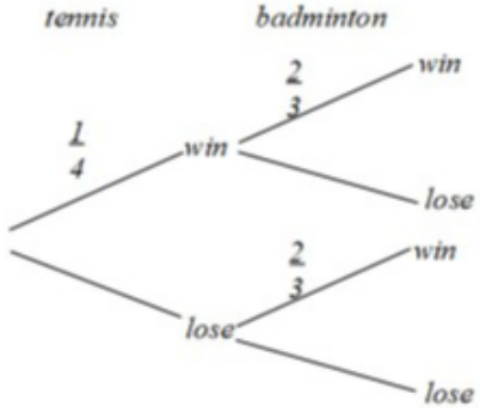
Johan is playing a tennis match and then a badminton match.
The probability he will win the tennis match is $\frac{1}{4}$. The probability he will win the badminton match is $\frac{2}{3}$.
What is the probability that he will LOSE both matches?

A. $\frac{3}{7}$

B. $\frac{4}{7}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: 3/4 × 1/3 = ¼ using the fact that the probability of losing is 1 minus the probability of winning and that the probability of two events both happening is the product of the probabilities.
Possible misconceptions: Learners may not understand tree diagrams and how to use them.
https://diagnosticquestions.com

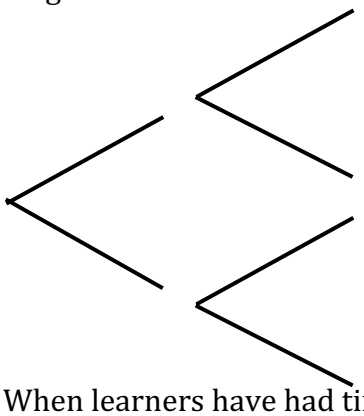
Generic competences

In doing this activity students will have an opportunity to develop the skill of representing information in a tree diagram.

Suggestions for teaching

This could be a **1-2-4-More** or **Think-Pair-Share** lesson. Give the problem to the learners to read individually, and to interpret for themselves, to give them practice in interpreting and using the information that they are given as they are required to do in exams.

After a few minutes ask learners how they would tackle the problem. Using their responses, make sure that everyone understands that they must draw a tree diagram and use the information given, and the answers the following questions, to label the tree diagram.



What is the probability that you will choose an envelope with 5 red and 3 blue?

What is the probability that you will choose an envelope with 2 red and 6 blue?

If you have chosen one type of envelope rather than the other type what difference does it make to the probability of choosing a sheet of red paper from it?

When learners have had time to draw and label a tree diagram and calculate an answer, ask the learners to work with a partner to explain their reasoning, to help learners who are struggling, and also to check their working and answers.

Then ask the learners to work in groups of four to compare answers. If there are groups who are still in difficulties, ask one learner from a group who are confident about the problem to swop with one learner from the group who are struggling to help them. is a It is a good learning experience to explain the concepts and methods to other learners as it develops reasoning and communication skills.

You will need about 15 minutes at the end of the lesson to get a pair of representatives from one or two groups to go to the board and explain their working to the class. One learner could label the tree diagram while the other learner explains the method.

Then repeat the explanation and summarise what has been learned.

Key questions

- What is the probability that you will choose an envelope with 5 red and 3 blue?
- What is the probability that you will choose an envelope with 2 red and 6 blue?
- If you have chosen one type of envelope rather than the other type what difference does it make to the probability of choosing a sheet of red paper from it?

An easier preliminary question

Same sweets https://aiminghigh.aimssec.ac.za/same-sweets/
which involves listing all the possible outcomes

Follow up

- **Lower Secondary**
  Same Birth Month https://aiminghigh.aimssec.ac.za/same-birth-month/
  In The Bag https://aiminghigh.aimssec.ac.za/grades-in-the-bag/
- **Upper Secondary** **Same Birthday**
  https://aiminghigh.aimssec.ac.za/same-birthday/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/MathsToys/videos

Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13