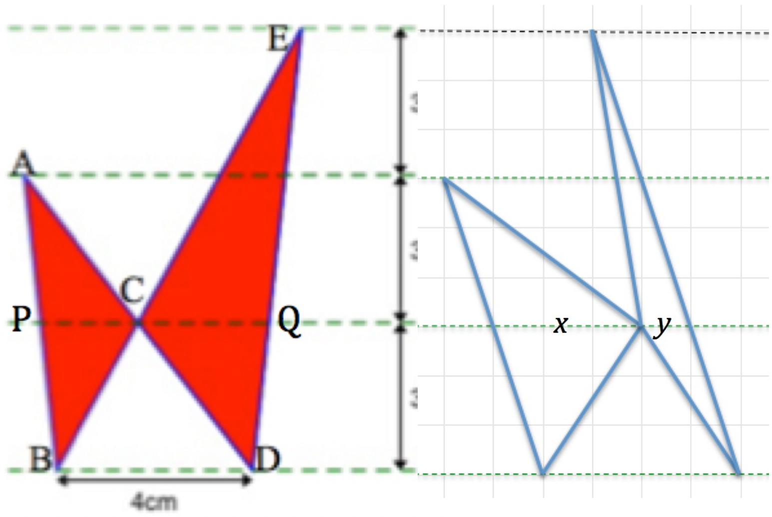
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

KISSING TRIANGLES



If ACD and BCE are straight lines find the total area of the two triangles by two different methods.

What are the lengths PC and CQ?

The triangles are re-drawn keeping the total area the same and keeping the distances between the green parallel lines as 3 cm and BD = 4 cm.

What is the value of y if x = 3 cm?

What other values can x and y take if the area is always the same?

SOLUTION

ADDITION METHOD

By similar triangles ΔAPC and ΔABD, PC = 2 cm so Area ABC = Area APC + Area BPC = 3 + 3 = 6 cm^2.

By similar triangles ΔECQ and ΔEBD, CQ = ⅔ × 4 = 2⅔ cm

Area CDE = Area CQE + Area CQD
= ½×6×2⅔ + ½×3×2⅔
= 8 + 4 = 12 cm^2.

Total area ABC + CDE = 18 cm^2

SUBTRACTION METHOD

Total area of triangles = Area BAD + Area BED – 2Area BCD
= 12 + 18 - 2×6
= 18 cm^2

If PC = x cm then area ΔPCA = area ΔPCB = 3x/2 so area ΔABC = 3x cm^2.
If CQ = y cm then area ΔCQE = 6y/2 = 3y and ΔCQD = 3y/2 so area ΔCDE = 9y/2 cm^2.

As the area is 18 we have the equation $3x + 9y/2 = 18$
which simplifies to $2x + 3y = 12$.
So if x = 3 then y = 2.

We now have 2 solutions (2, 8/3) and also (3, 2) but there are infinitely many solutions corresponding to points on the line $2x + 3y = 12$.

NOTES FOR TEACHERS

Why do this activity?

This activity makes use similar triangles and of the theorem that a line drawn parallel to one side of a triangle divides the other two sides proportionally. It reinforces ideas of area and develops learners' problem solving skills by asking them to find the area by two different methods. The activity focuses on Euclidean Geometry but leads naturally to the use of algebra and the concept of a linear function of two variables that has as solutions the coordinates of points on a straight line. These are all concepts that the learners are already familiar with and the activity makes connections between them.

Intended learning outcomes

- Investigation into line segments joining points that divide the sides of triangles in the same ratio producing similar triangles.
- Development of familiarity with the use of similar triangles and the theorem that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and in this example the Mid-point Theorem for ΔBAD as a special case);
- Development of problem solving skills.
- Review of the methods for calculating area.

Possible approach

You might write the question on the board or print the top part of page 1 as a worksheet.

Let the learners work in pairs to find the area. Then ask the pairs to discuss their solutions in groups of 4 and to try to find at least two different methods between them.

Then ask pairs if learners to explain their methods to the class.

Discuss what happens when the diagram is changed keeping the area the same and ask learners to find y when x = 3.

In the plenary ask learners to suggest other pairs of values of x and y that give the same area. Ask 'how many solutions are there? Discuss the linear relation between x and y and that there are infinitely many solutions corresponding to the points on a straight line.

Review the theorem about line segments joining the midpoints of two sides of a triangle and relate it to ΔBAD in this activity where BCD is a straight line. Discuss the similar triangles in ΔBED, where BCE is a straight line, and discuss how this problem uses line segments joining points that divide the sides in the same ratio producing similar triangles.

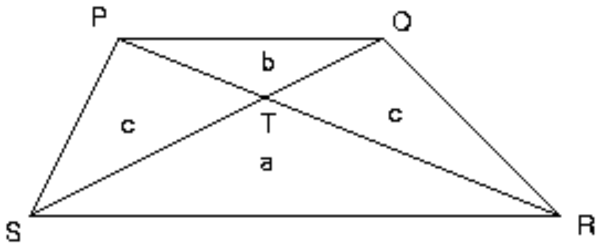
Key Questions

What is the height of that triangle?
Can you spot any similar triangles?
What can you say about the triangles APC and ABD?
Can you find the length of PC?
What can you say about the triangles ECQ and EBD?
Can you find the length of CQ?
Which areas can you find?
Can you calculate the areas by another method?
What is the same and what is different about the two sketches of the kissing triangles?
What values of x and y can you find which make the total area of the triangles the same?

Possible support

To help learner to get started you could suggest that they draw the diagram for themselves on squared paper and mark in the areas that they can find.

Possible extension



A trapezium is divided into four triangles by its diagonals. Suppose the two triangles containing the parallel sides have areas *a* and *b*.

Prove that the total area of the trapezium is $a + b + 2\sqrt{ab}$