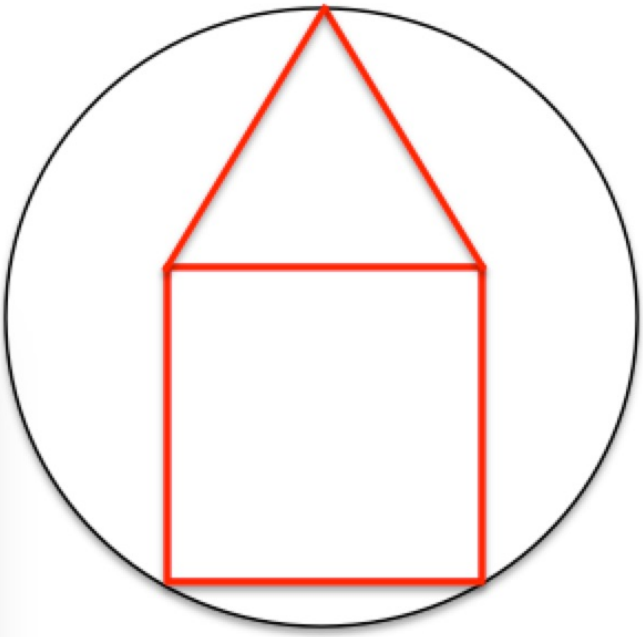
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

ENCIRCLING

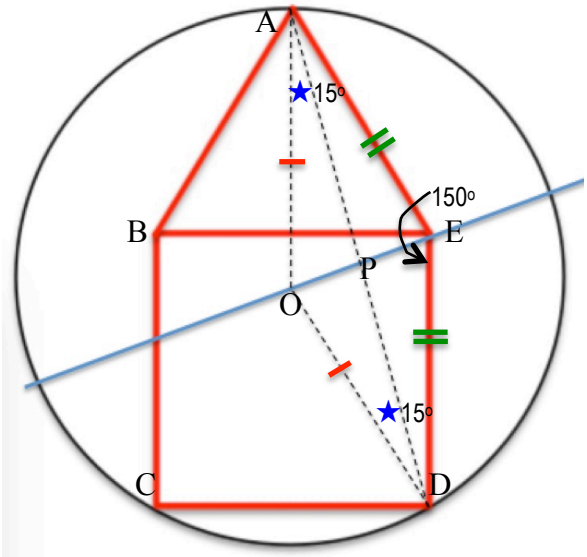


This shape is an equilateral triangle sitting on top of a square.

What is the radius of the circle that circumscribes the shape?

Hint: join the centre to the vertex at the top of the triangle and to one of the lower vertices of square.

SOLUTION



Construct the diameter through E.
Join centre O to points A and D.
Join AD.

As ΔABE is equilateral AO is perpendicular to BE and $\angle OAE = 30^o$.

As angle $\angle AED = 60^o + 90^o = 150^o$ and ΔAED is isosceles
$\angle DAE = \angle ADE = 15^o$

$\angle OAP = \angle OAE - \angle PAE = 30^o - 15^o = 15^o$

ΔAOD is isosceles (OA=OD) so $\angle ODP = 15^o$ and ΔOAE is congruent to ΔODE (SSS).
Hence AD is the internal bisector of ∠sOAE and ODE and AD is an axis of symmetry of the quadrilateral OAED.

So OAED is a kite with OE and AD as axes of symmetry and the radius of the circle OA is equal to the lengths of the edges of the original equilateral triangle and square.

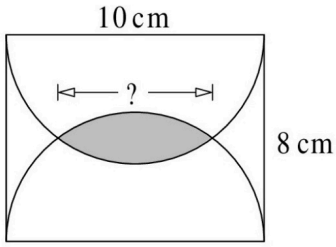
NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Two semicircles are drawn in a rectangle as shown.

What is the width of the overlap of the two semicircles?



A 3 cm B 4 cm C 5 cm D 6 cm

D. is the correct answer.
Draw a 3-4-5 triangle to show that half this width is 3 cm.

Common Misconceptions

A. Some learners may have found the half width to be 3 cm and then forgotten to double it.

B. Perhaps some learners just made a guess but others might have thought "I think 4 cm because 10 minus 8 is 2 and 2 times 2 is 4".

C. Learners may have thought "it looks about 5cm compared to the 10cm at the top".

https://diagnosticquestions.com

Why do this activity?

This activity gives learners practice in mathematical (geometrical) reasoning. It might surprise learners to be asked for a length when they are not given any lengths so they have to think carefully about the information given and decide to shape their own problem to be solved. This is an important skill in problem solving in general. Actually learners need to recognise that they are given the information that all the edges of the equilateral triangle and the square are the **same** length. Solving this problem only requires basic knowledge of the properties of equilateral triangles, squares, isosceles triangles and symmetry and not knowledge of the circle theorems.

Intended learning outcomes

Practice in geometrical reasoning and proof.

Possible approach

Start with the diagnostic question.

Then you could draw the diagram on the board and ask the learners to find the radius of the circle. You could allow learners to work in pairs and to discuss their ideas.

You might have copies of the hint diagram that you can give to learners who are struggling or you might ask key questions to help the learners to get started. Don't tell them what to do, make sure instead that you just ask questions.

After a while you might draw the hint diagram on the board.

When at least half of the learners have found the solution you could ask one of a pair of learners who have found the solution to swap places with one of a pair who have not found a solution and then to explain it to their new partner.

Finally ask a pair of learners to explain the solution, one writing on the board and the other one giving reasons for each step. At the end summarize what geometrical facts have been used.

Key questions

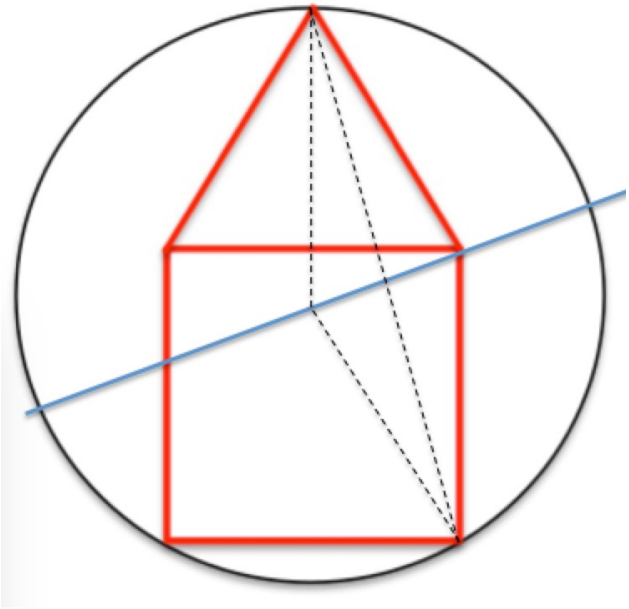
Do you see any symmetry in the original diagram? Can you explain it?
Can you mark all the angles you know in the diagram?
Can you mark equal lengths in the diagram?
What angles do you know in the original diagram?
What do you know about the lengths in the original diagram? Why?
How big is angle ∠AED? Why?
Could you use a diameter through one of the vertices of the square if you put one in?
What can you say about triangle ΔAED? Why?
What do you know about OA? Why?
What is angle ∠OAE? Why?
What is angle ∠DAE? Why?
Can you find ∠EDA? Why?
Can you find angles ∠OAD and ∠OAD?
What can you say about the quadrilateral OAED?

Possible extension

See Geoprob https://aiminghigh.aimssec.ac.za/grades-10-to-12-geoprob/

Possible support

Use this hint



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Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**