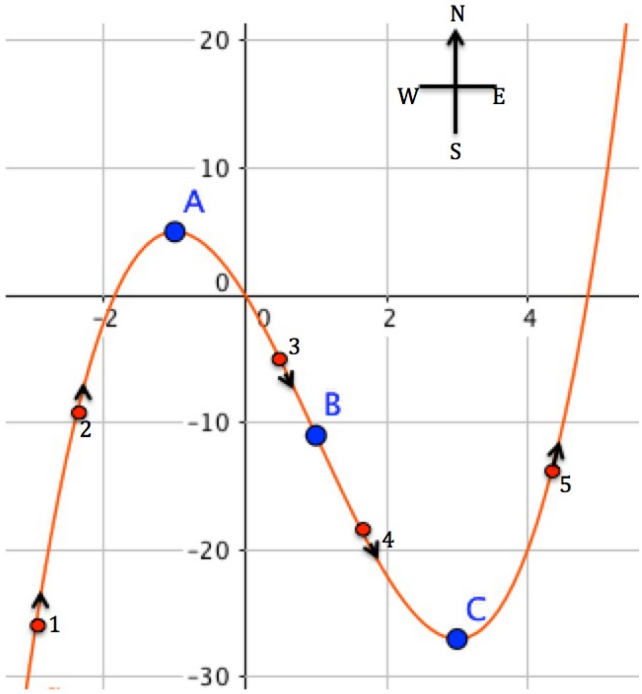
D'RIVING



Imagine this graph is a road and you are driving from the bottom left hand (South West) corner to the top right hand (North East) corner. Describe how you would steer the car.

When would you be turning the steering wheel to the right?

When would you be turning the steering wheel to the left?

When would you be driving straight ahead?

Now look at the graph again. How sharp were those bends? Thinking nor of the graph, what was happening to the gradient when you were steering to the right?

What was happening to the gradient when you were steering to the left.

What was happening to the gradient when you were steering straight ahead?

What was the gradient when you were travelling due East?

	1	2	A	3	B	4	C	5
Gradient	+	+	0	–	–	–	0	+
First derivative	+	+	0	–	–	–	0	+
Gradient increasing **I** or decreasing **D**	D	D	D	D	*	I	I	I
Second derivative	–	–	–	-	0	+	+	+

This is the graph of $f(x) = x^3 - 3x^2 - 9x$.

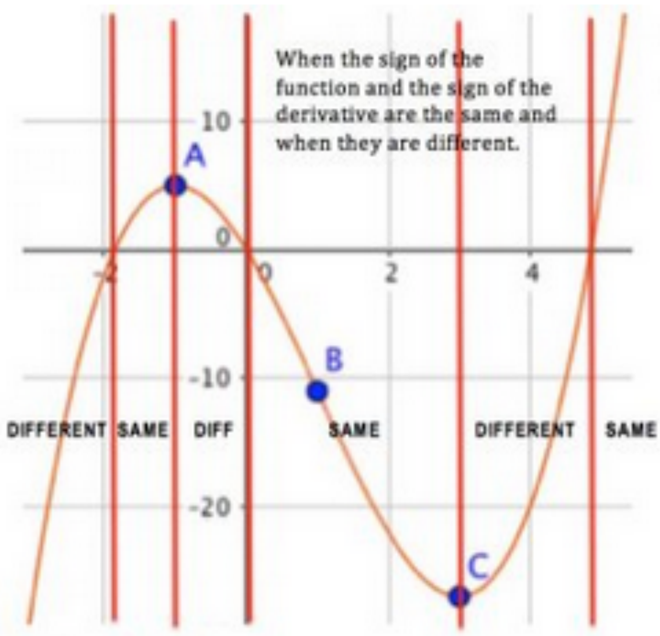
Find the first derivative and the second derivative of this function and the coordinates of points A, B and C.

When do this cubic function and its first derivative have the same sign?

Sketch the graph of a cubic with only one turning point.

Sketch graphs of straight lines and quadratic functions.

Describe when these functions and their first derivatives have the same sign and when they have different signs.



HELP

Visualisation is the key to advanced mathematical thinking. It engages different parts of the brain in making connections. You might visualise drawing of the curve by your car so that, as you describe how you would steer the car along the path, you are describing how to draw the curve.

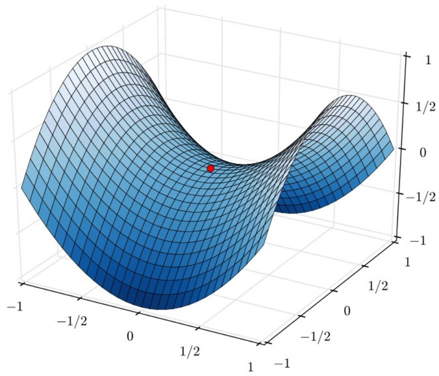
NEXT

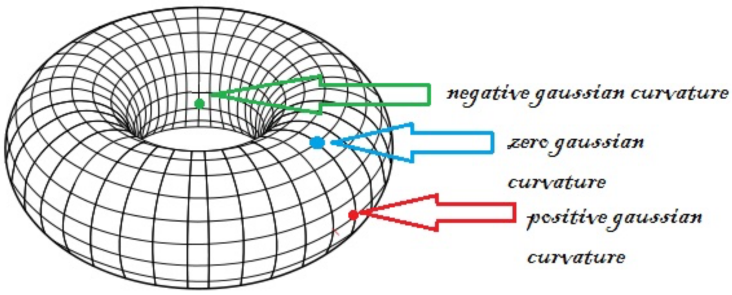
Describe how you would drive over this mountain pass in Lesotho.

See the blue image below for the mathematical view of a saddle point.

Try to create analogies for other concepts in calculus. This is very open ended but will really get you thinking about calculus as the mathematics of rates of change.

If you go on studying maths, and that means a lot more calculus, you'll learn about curvature of surfaces. A sphere has positive curvature. A flat surface has zero curvature. Some surfaces, like a saddle and a torus have both positive and negative curvature.





Sign of the derivative of a function at each point
The derivative of the function is positive when travelling towards the north, negative when travelling towards the south.

Sign of the second derivative of a function at each point
If your steering wheel is turned clockwise from the neutral position then the second derivative is negative. If it is turned anticlockwise from the neutral position then the second derivative at that point is positive.

Sign of the third derivative of a function at each point
If the steering wheel is in the process of turning in the anti clockwise direction then the third derivative is positive. If the steering wheel is in the process of turning in the clockwise direction then the third derivative is negative.

Differentiability condition at each point
The function is differentiable at points on the road when is it possible to drive along smoothly without having to suddenly turn the steering wheel.

Points of inflection
Points of inflection occur at the points, and only the points, where the steering wheel passes through the neutral position.

Note on terminology
The **'neutral position'** is the position of the steering wheel in which the car travels forwards in a straight line. A **clockwise turn** from this position causes the car to turn right and an **anticlockwise turn** from this position causes the car to turn left.

Cards with statements to discuss in relation to the D'RIVING graph.

Do you agree or disagree with these statements? Give reasons for your answers.

NOTES FOR TEACHERS

SOLUTION

You turn the steering wheel to the right from the start all the way to B because the gradient is decreasing.

You turn the steering wheel to the left all the way from B to the end because the gradient is increasing.

You drive straight ahead at B where the gradient is stays the same for an instant.

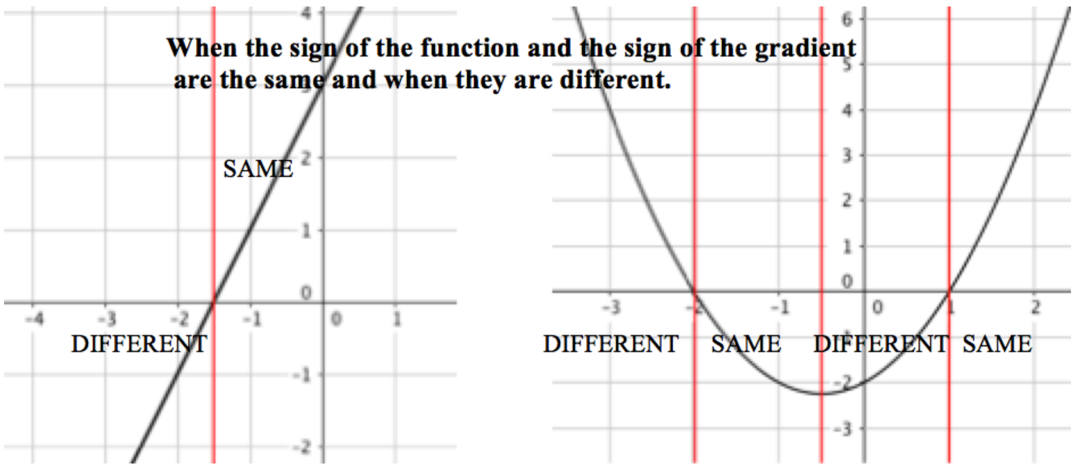
When travelling due East (in the positive x direction) the gradient is zero.

$f(x) = x^3 - 3x^2 - 9x$

The first derivative $f\,'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$
so the coordinates of the turning points A and C where the first derivative is zero are A(-1, 5) and CC3, -27).

The second derivative $f\,''(x) = 6x - 6$
so the coordinates of B where the second derivative is zero are B(1, -11).

This cubic function and its first derivative have the same sign when the function is positive and the gradient is positive and also when the function is negative and the gradient is negative.



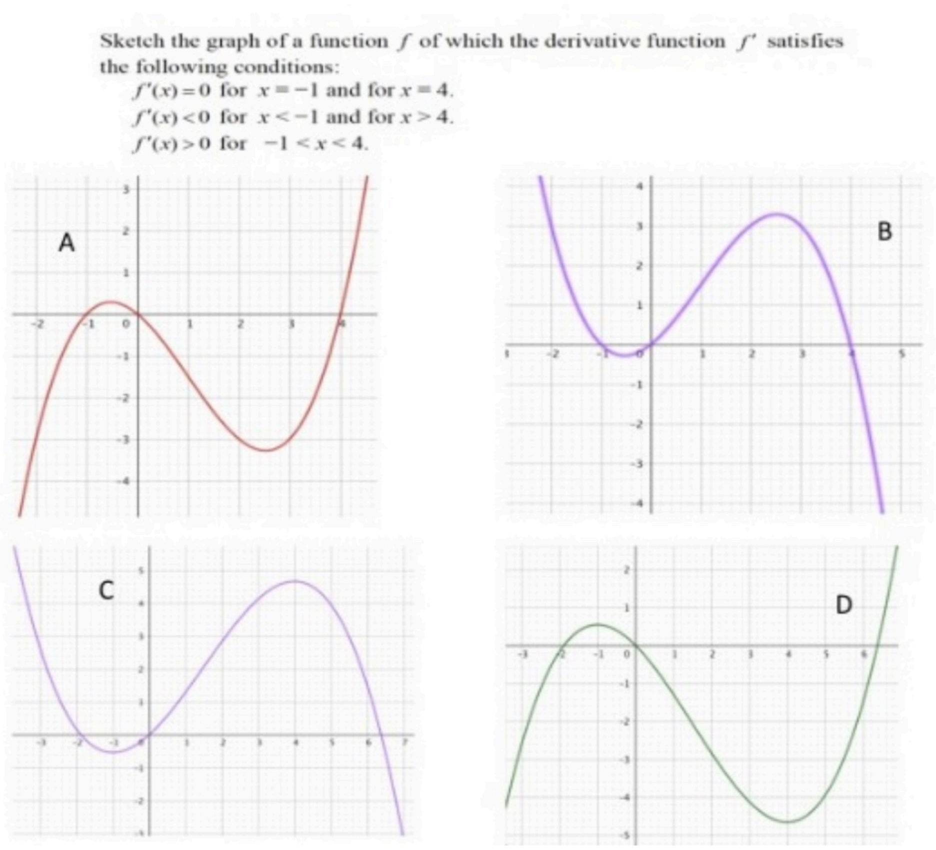
The vertical red lines separate the regions in which the sign of the function and the sign of the gradient are the same and when they are different.

Why do this activity?

This activity often proves incredibly useful to learners because having a sound geometrical visualisation for concepts in calculus is essential in any application. It also proves very useful in checking that calculations make sense and for uncovering misconceptions about calculus.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is C local maximum or minimum at x=-1 and x=4 for x<-1 and x>4 f(x) is decreasing and for -1 < x < 4 f(x) is increasing

For answers A, B and D students have misread the graphs looking at intercepts with x-axis instead of turning points and the sign of the function instead of the sign of the derivative.

https://diagnosticquestions.com

Learning objectives

In doing this activity students will have an opportunity to

- deepen understanding of functions, derivatives, turning points and gradients;
- explore the relationship of signs of functions and derivatives to gradients.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for Teaching

You might like to give out the cards on page 2 so that learners have clear statements to discuss.

This need not be a long activity and can be used at any point in the curriculum where the concepts in any of the 5 analogies have been encountered. You can focus on a couple of the most relevant analogies if desired.

You could simply set up the situation and let the students enter into discussion. Students can think about the ideas in small groups and sketch 'road maps' on which to test their ideas.

Alternatively, you can sketch a curve with, say, 4 turning points on the board and ask for a volunteer to model the motion of the imaginary steering wheel as you trace your finger along the curve. Another volunteer can record the motion of the steering wheel, paying particular attention to the direction or speed of turn. You could then sketch a more 'demanding' road and repeat the exercise.

There are at least three levels of approach to this problem:

1) Once students are intuitively clear as to which analogies are largely reliable the lesson can move on and the analogies can be referred to as a guide throughout subsequent study of calculus.

2) Students can try to construct convincing justification that the analogies are sound, including some thought on when the analogies break down (i.e. what sorts of roads do the analogies work for, and what sorts of 'pathological' roads do the examples not work for?)

3) Students might try to come up with some analogies of their own which others might test out. For example, other analogies for the sign of the gradient might involve mountains, valleys or hills.

Note that various misconceptions might be unearthed during this task, and many more advanced concepts in mathematics might be raised. Perhaps by the students themselves. See the possible support below for some of these.

Misconceptions or errors to look out for are:
1. The steeper the gradient the more the wheel needs to be turned
2. A function can be used to describe, say, a circle (No: A function is single valued)
3. A point of inflection must also be a stationary point (No: That is a stationary point of inflection)

Advanced concepts in mathematics which might be raised in some form are:
1. What is a function as opposed to a curve?
2. What is a continuous / differentiable function?
3. What is curvature?
4. Are there functions that are only twice differentiable?

Key questions

- Who can describe the motion of a wheel through a journey?
- Can you imagine driving along the road indicated on this map?
- For what sorts of crazy curves might these analogies not work?
- Can you give a clear justification for you results (using words or algebra)?
- What can we say about a car which is moving due north at some point?

Follow up

Derivative Matching https://aiminghigh.aimssec.ac.za/year-12-derivative-matching/