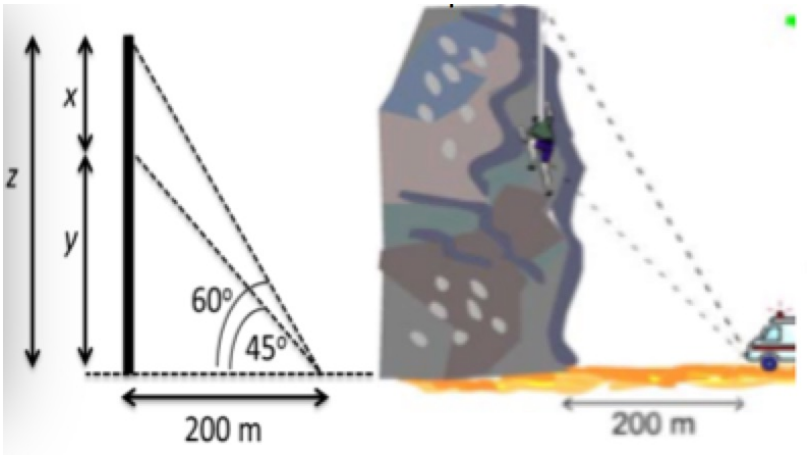
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

CLIFF RESCUE



A climber is stuck at the middle of a cliff. A rescue worker on the ground is 200 m from the bottom of the cliff. The angles of elevation of the climber and of the top of the cliff as seen by the rescuer are 45^o and 60^0 respectively.

Draw a diagram.

Another rescuer at the top of the cliff is going to lower a rope to the climber but he is not sure that his rope is long enough.

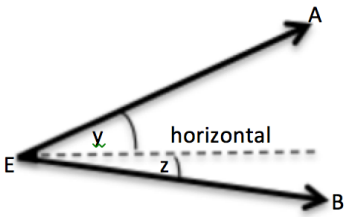
What length of rope should he use to save the climber?

Help

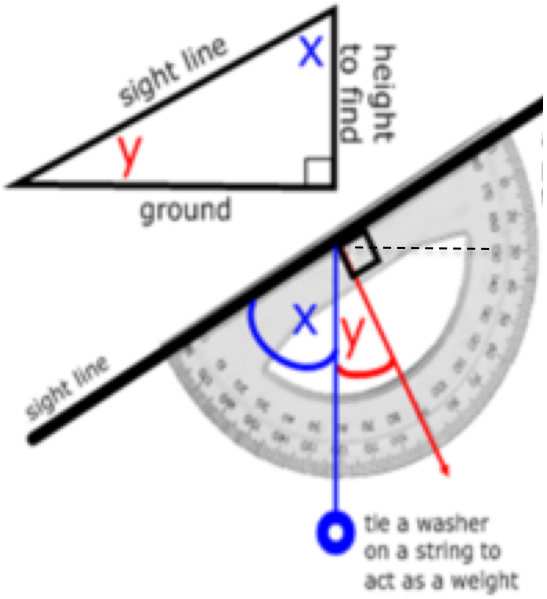
Making your own clinometer gives you a better understanding of angles of elevation and depression. In this problem you use a right angled triangle and a trig ratio to calculate the length z metres. Then you use another right angled triangle to find the length y metres.

When you know y and z you can find the length x. The rope needs to be a bit longer than x as shown in the diagram because the rescuer and the climber both need to be able to get hold of it.

Make your own clinometer to measure angles of elevation and depression



If an observer with her eye at E looks up to a point A, then the angle between the horizontal and EA, shown in the diagram as ∠y, is called the angle of elevation. Similarly if the observer looks down to a point B then the angle between the horizontal and EB, shown as ∠Z, is called the angle of depression.



The diagram shows an instrument to measure these angles.

It can be made simply and cheaply. To read the angles you need a protractor. To get an accurate vertical you need a weight (such as a heavy paperclip, a washer or a ball of plasticene) tied to a piece of string as the plumb line. To make the sight line use a drinking straw or stick.

If you hold the protractor with the 90^o mark on the vertical the sight line is horizontal. Two people find the angle of elevation ∠y, one holds the instrument to their eye looking at the object of interest through the straw along the line of sight. The other person reads the angle between the vertical plumb line and the 90^o mark on the protractor. This is ∠y equal to the angle of elevation.

Extension

Make a clinometer and make up some challenges for yourself about how you could use it.

NOTES FOR TEACHERS

SOLUTION

$$\tan 45° = \frac{y}{200}$$

$$1 = \frac{y}{200}$$ which gives $y = 200$ as you have known!

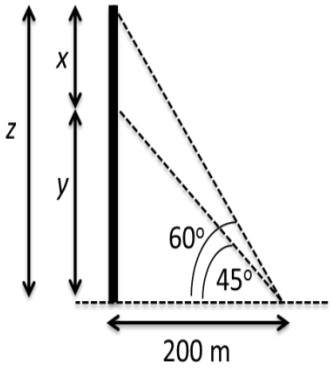
$$\tan 60° = \frac{z}{200}$$

$$\sqrt{3} = \frac{z}{200}$$

$$z = 200\sqrt{3}$$

$$x = z - y = 200(\sqrt{3} - 1)$$

$$x = 146.4\ m$$

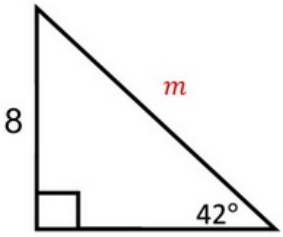


To be sure to have sufficient rope a length of at least 150 metres is needed.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Which of the following would correctly calculate the length of side m?



A: $\sin(42) \times 8$

B: $\sin(42 \times 8)$

C: $\frac{8}{\sin(42)}$

D: $\frac{\sin(42)}{8}$

The correct answer is **C**.

Learners giving other answers know that they should use the sine ratio but do not understand how to use it.

https://diagnosticquestions.com

Why do this activity?

This application shows learners an application of school mathematics to a real life problem.

Learning objectives

In doing this activity students will have an opportunity to:

- practice reading a question and drawing a diagram from the information given;
- use and consolidate knowledge of special angles;
- develop problem solving skills.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically and flexibly**, reason logically and give explanations;
- **visualize** and develop the skill of interpreting visual images representing concepts and situations;
- interpret information and **solve a practical problem.**

Suggestions for teaching

Give learners the problem to read and ask them to draw the diagram for themselves. You could pose the problem in a rather more challenging way by not telling the learners that the rescue worker on the ground is 200 m from the base of the cliff. Rather simply ask for suggestions as to how he could find the height of the cliff and the height of the climber and what other information he would need to find out in order to do so.

Review the diagrams. Then ask the learners to work individually to solve the problem then to compare their answers with a partner.

Finally ask a pair of learners to write up the solution on the board and to explain their working.

Emphasise that it is much quicker to use the tangent ratios that that they should know for 45^o and 60^o than to use a calculator.

Discuss the practical situation and why a slightly longer rope is needed.

Key Questions

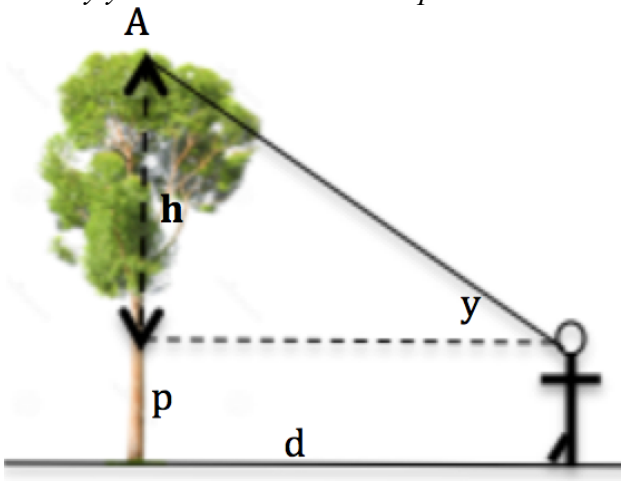
- Can you find right angled triangles?
- Which lengths can you calculate from the right angled triangles.
- How would the rescue worker on the ground find the height of the cliff?
- How would the rescue worker on the ground find the height of the climber?

Follow up

Similar activities:
Cable Car https://aiminghigh.aimssec.ac.za/year-10-cablecar/
At Sea https://aiminghigh.aimssec.ac.za/years-10-11-at-sea/

Some suggestions for practical activities, possibly to do before questions like Cliff Rescue.
As a preparation for using trigonometry for finding angles and distances you can ask learners to draw accurate scale drawings and make measurements from their drawings to find angles and distances. All these activities are suitable for lower secondary students. Older learners can do the same activities; using sketches rather than accurate scale drawing they can find the angles and distances by trigonometry.
Make a clinometer to measure angles of elevation and depression (See Learner Worksheet page 1).

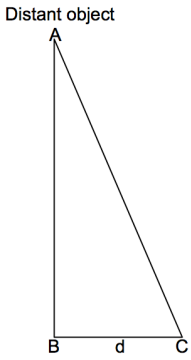
Measuring the height of trees and buildings
Clearly you can't climb to the top to do the measurement so scale drawing or trigonometry is useful.



1. Measure a distance along the ground (say 30 metres) from the bottom of the tree or building and stand on this spot. Call this distance d
2. Measure the angle between the horizontal and the line from your eye to the top of the tree (angle of elevation). Call this angle y.
3. Draw the right angled triangle with a horizontal line from your eye to the tree, a vertical line up the tree (or building) and the line from your eye to the top.
4. Mark the distance d, the height of the building above this level h and angle y.
5. Then h divided by d is the tangent of angle y. Use trigonometry to calculate h. Add the height p from the ground to your eye level to get the height of the tree.

How far is it across the river or how far is that boat away across the sea

Can you find this without swimming across? You might do this across some imaginary river say in your school hall or playground or a nearby field.



1. You want to find the distance straight across the river. Take a sighting on a tree or some object directly opposite to you on the far bank, call this object A. Now mark the spot you are standing with some object B that you will be able to see from along the river bank. (It might be a far away village, or a ship at sea and you might be on a beach instead of on a river bank.)
2. Measure the distance d between where you are standing and another spot C along the river bank on your side of the river. ABC is a right angled triangle.
3. From the second observation point C measure the angle between the direction CB and the direction CA. Then the distance AB divided by the distance BC = d (which you know) gives you the tangent of this angle ∠CAB and you can calculate the width of the river.

East West and North South distances

1. Village A is 50 kilometres North West of village B. How far North is it from village B and how far west is it?
2. The diagram you need is a right angled triangle with angles of 45 degrees and hypotenuse of length 50 km. You can use sine and cosine to calculate the EW and NS distances or Pythagoras Theorem.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**