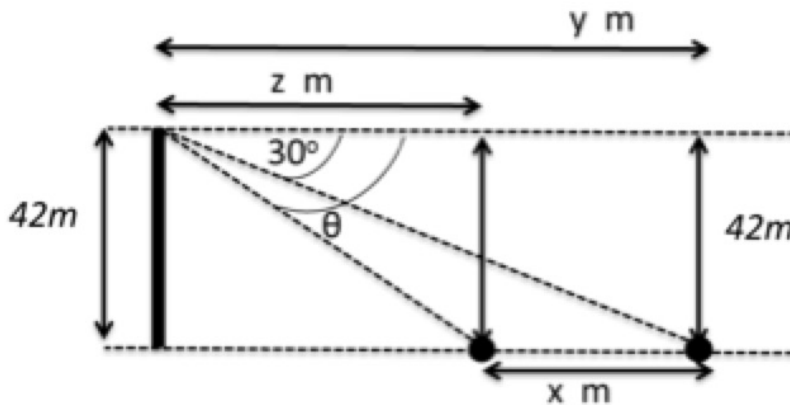# CLIFF RESCUE

Two ships are heading towards a lighthouse, on the same path, one behind the other. From a height of 42 metre the closer ship can be observed at an angle of depression θ where $\tan \theta = \frac{4}{5}$ and the other ship at an angle of depression of 30 degrees. Draw a diagram.

How far are the two ships from each other?

Help
You could make your own clinometers and do some projects.

Make your own clinometer to measure angles of elevation and depression
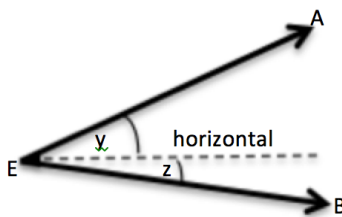
If an observer with her eye at E looks **up** to a point A, then the angle between the horizontal and EA, shown in the diagram as ∠y, is called the angle of elevation. Similarly if the observer looks **down** to a point B then the angle between the horizontal and EB, shown as ∠Z, is called the angle of depression.
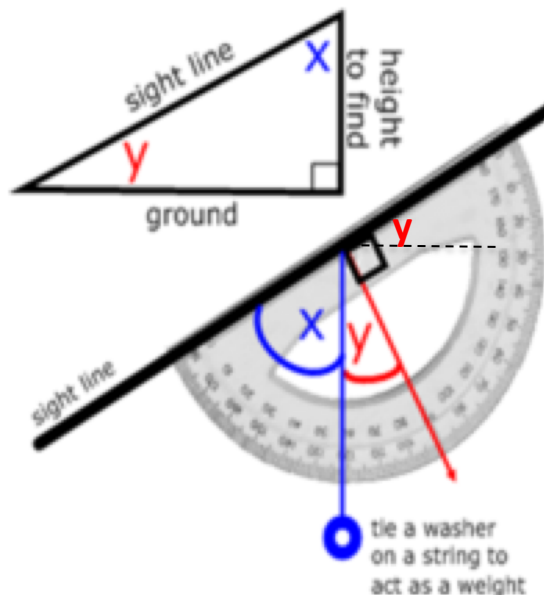
The diagram shows an instrument to measure these angles. It can be made simply and cheaply. To read the angles you need a protractor. To get an accurate vertical you need a weight (such as a heavy paperclip, a washer or a ball of plasticene) tied to a piece of string as the plumb line. To make the sight line use a drinking straw or stick.

If you hold the protractor with the 90° mark on the vertical the sight line is horizontal. You need two people to find the angle of elevation ∠y, one person holds the instrument to their eye looking at the object of interest through the straw along the line of sight. The other person reads the angle between the vertical plumb line and the 90° mark on the protractor. This is ∠y equal to the angle of elevation.

Extension
Try the similar problem Cliff Rescue https://aiminghigh.aimssec.ac.za/grades-10-and-11-cliff-rescue/

NOTES FOR TEACHERS

SOLUTION

$$\tan \theta = \frac{42}{z} = \frac{4}{5}$$

$$z = \frac{5 \times 42}{4} = 52.5$$

$$\tan 30 = \frac{42}{y}$$
$$y = 42\sqrt{3}$$

The distance between the ships is

$$x = y - z = 42\sqrt{3} - 52.5 = 20.25m$$

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. **Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.**
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

What is the length of the shadow a 45m tower casts when the angle of elevation to the sun is 32°?

Calculators are needed for this question

*picture not to scale
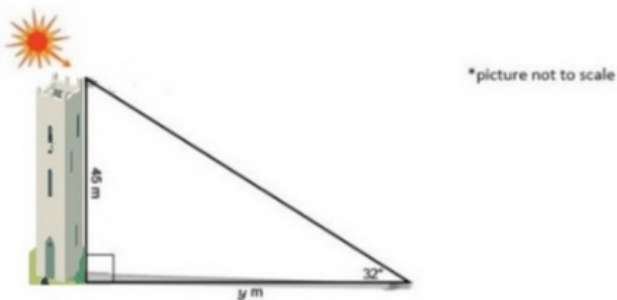
A — 28m   B — 38m   C — 55m   D — 72m

The correct answer is **D.** $\tan 32^o = 45/y$

Possible misconceptions:

A. Students have calculated $45*\tan 32^o$

B. Students have calculated $45*\cos 32^o$

C. Students have calculated $\tan^{-1}(45/32)$

https://diagnosticquestions.com

2

Why do this activity?
This application shows learners an application of school mathematics to a real life problem.

Learning objectives
In doing this activity students will have an opportunity to:
- practice reading a question and drawing a diagram from the information given;
- use and consolidation of knowledge of special angles.
- Develop problem solving skills.

Generic competences
We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.
In doing this activity students will have an opportunity to:
- **think mathematically** to apply knowledge and skills;
- **visualization** - develop the skill of interpreting and creating visual images to represent concepts and situations;
- **i**nterpret and **solve problems** in a variety of situations.

Suggestions for teaching
Start with the diagnostic quiz to remind learners of what they have learned about the tangent ratio in a right angled triangle and about angles of elevation and depression.

Give learners the problem to read and ask them to draw the diagram for themselves.

Review the diagrams. Then ask the learners to work individually to solve the problem then to compare their answers with a partner.

Finally ask a pair of learners to write up the solution on the board and to explain their working.

Emphasise that it is much quicker to use the tangent ratios that that they should know for 30° than to use a calculator.

Discuss the practical situation and why it is important for navigators to be able to calculate distances between ships.

Key questions
- Can you find right angled triangles?
- Which distances can you calculate from the right angled triangles?

Follow up
Some suggestions for practical activities
As a preparation for using trigonometry for finding angles and distances you can ask learners to draw accurate scale drawings and make measurements from their drawings to find angles and distances. All these activities are suitable for lower secondary classes. Older learners can do the same activities and find the angles and distances by trigonometry using sketches rather than accurate scale drawing,.

Measuring the height of trees and buildings
*Clearly you can't climb to the top to do the measurement so scale drawing or trigonometry is useful.*
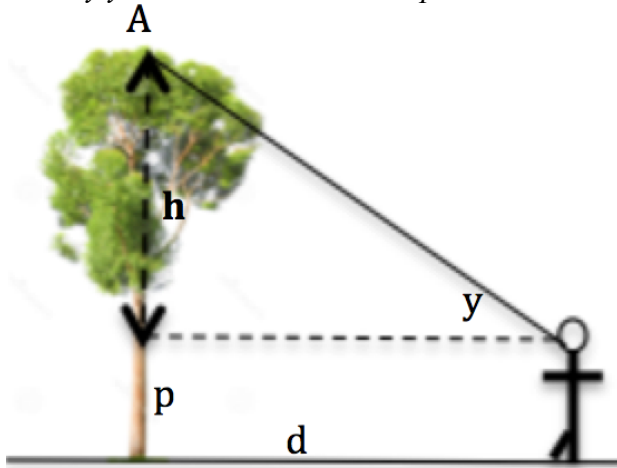
1. Measure a distance along the ground (say 30 metres) from the bottom of the tree or building and stand on this spot. Call this distance d
2. Measure the angle between the horizontal and the line from your eye to the top of the tree (angle of elevation). Call this angle y.
3. Draw the right angled triangle with a horizontal line from your eye to the tree, a vertical line up the tree (or building) and the line from your eye to the top.
4. Mark the distance d, the height of the building above this level h and angle y.
5. Then h divided by d is the tangent of angle y. Use trigonometry to calculate h. Add the height p from the ground to your eye level to get the height of the tree.

How far is it across the river or how far is that boat away across the sea
Can you find this without swimming across? You might do this across some imaginary river say in your school hall or playground or a nearby field.

1. You want to find the distance straight across the river. Take a sighting on a tree or some object directly opposite to you on the far bank, call this object A. Now mark the spot you are standing with some object B that you will be able to see from along the river bank. (It might be a far away village, or a ship at sea and you might be on a beach instead of on a river bank.)
2. Measure the distance d between where you are standing and another spot C along the river bank on your side of the river. ABC is a right angled triangle.
3. From the second observation point C measure the angle between the direction CB and the direction CA. Then the distance AB divided by the distance BC = d (which you know) gives you the tangent of this angle ∠CAB and you can calculate the width of the river.
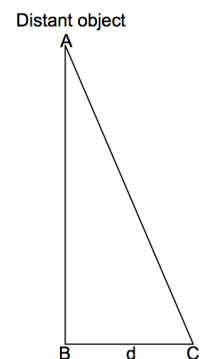
East West and North South distances
1. Village A is 50 kilometres North West of village B. How far North is it from village B and how far west is it?
2. The diagram you need is a right angled triangle with angles of 45 degrees and hypotenuse of length 50 km. You can use sine and cosine to calculate the EW and NS distances or Pythagoras Theorem.