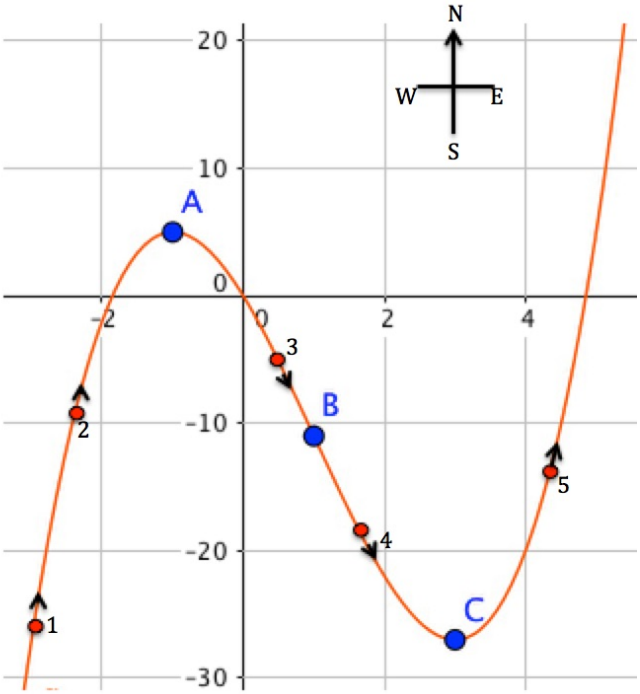
D'RIVING



Imagine this graph is a road and you are driving from the bottom left hand (South West) corner to the top right hand (North East) corner. Describe how you would steer the car.

When would you be turning the steering wheel to the right?

When would you be turning the steering wheel to the left?

When would you be driving straight ahead?

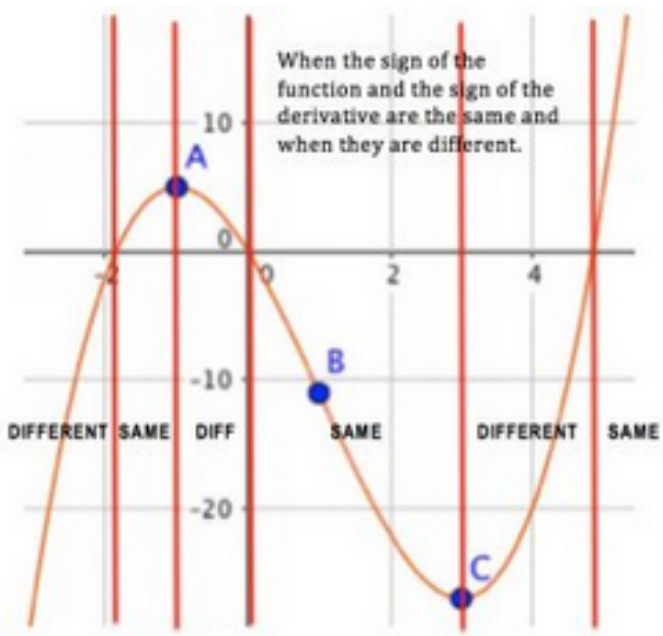
Now look at the graph again. How sharp were those bends? Thinking nor of the graph, what was happening to the gradient when you were steering to the right?

What was happening to the gradient when you were steering to the left.

What was happening to the gradient when you were steering straight ahead?

What was the gradient when you were travelling due East?

	1	2	A	3	B	4	C	5
Gradient	+	+	0	–	–	–	0	+
First derivative	+	+	0	–	–	–	0	+
Gradient increasing **I** or decreasing **D**	D	D	D	D	*	I	I	I
Second derivative	–	–	–	-	0	+	+	+



This is the graph of $f(x) = x^3 - 3x^2 - 9x$.

Find the first derivative and the second derivative of this function and the coordinates of points A, B and C.

When do this cubic function and its first derivative have the same sign?

Sketch the graph of a cubic with only one turning point.

Sketch graphs of straight lines and quadratic functions.

Describe when these functions and their first derivatives have the same sign and when they have different signs.

HELP

Visualisation is the key to advanced mathematical thinking. It engages different parts of the brain in making connections. You might visualise your car actually drawing the curve. As you describe how you would steer the car along the path, you are describing how to draw the curve.

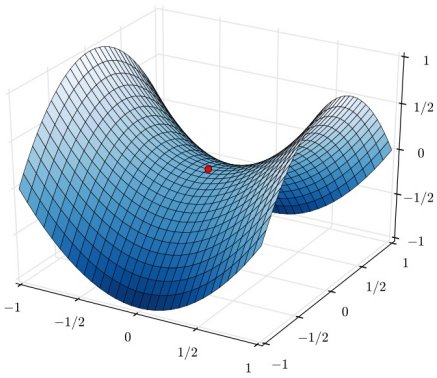
NEXT

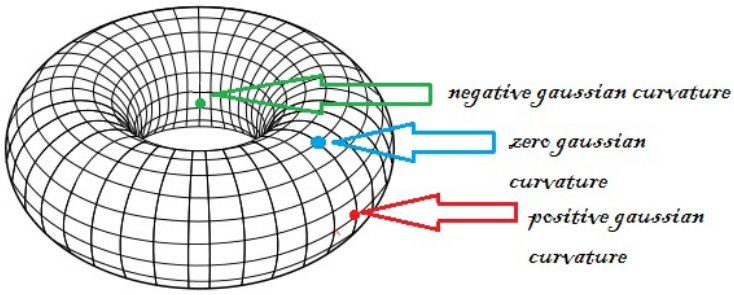
Describe how you would drive over this mountain pass in Lesotho.

See the blue image below for the mathematical view of a saddle point.

Try to create analogies for other concepts in calculus. This is very open ended but will really get you thinking about calculus as the mathematics of rates of change.

If you go on studying maths, and that means a lot more calculus, you'll learn about curvature of surfaces. A sphere has positive curvature. A flat surface has zero curvature. Some surfaces, like a saddle and a torus have both positive and negative curvature at different points of the surface.





If you have a computer connected to the internet then download for free the brilliant easy-to-learn software MSW Logo **https://mswlogo.en.softonic.com/download**. Written about 50 years ago as an educational tool, nothing better in the way of simple do-it-yourself educational software has been created since. The principle of steering along different paths is what underlies Logo.

See **First Forward Into Logo** on the NRICH website for a series that introduces Logo programming for beginners. The 12 challenges are written for you to work through at your own pace. You don't need to know anything about coding (programming) at the start: you will learn enough at the first stage to write your own programs, and you may surprise yourself by how much you can achieve right away.

Sign of the derivative of a function at each point
The derivative of the function is positive when travelling towards the north, negative when travelling towards the south.

Sign of the second derivative of a function at each point
If your steering wheel is turned clockwise from the neutral position then the second derivative is negative. If it is turned anticlockwise from the neutral position then the second derivative at that point is positive.

Sign of the third derivative of a function at each point
If the steering wheel is in the process of turning in the anti clockwise direction then the third derivative is positive. If the steering wheel is in the process of turning in the clockwise direction then the third derivative is negative.

Differentiability condition at each point
The function is differentiable at points on the road when is it possible to drive along smoothly without having to suddenly turn the steering wheel.

Points of inflection
Points of inflection occur at the points, and only the points, where the steering wheel passes through the neutral position.

Note on terminology
The **'neutral position'** is the position of the steering wheel in which the car travels forwards in a straight line. A **clockwise turn** from this position causes the car to turn right and an **anticlockwise turn** from this position causes the car to turn left.

Cards with statements to discuss in relation to the D'RIVING graph.

Do you agree or disagree with these statements?

Give reasons for your answers.