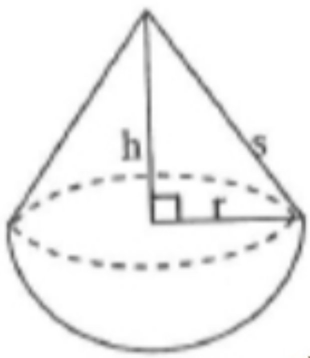
TOPS



For your company, you have to design spinning tops made from two pieces glued together, a hemisphere and a cone' and plan how they will be manufactured, packaged and sold.

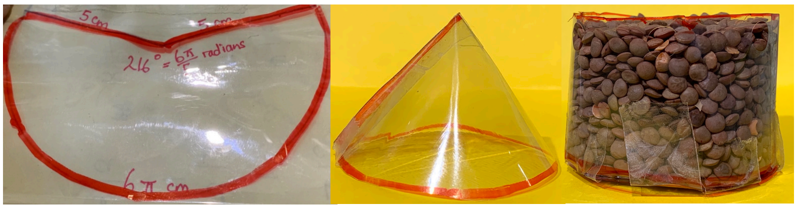
(1) If the radius of the circular surface where the pieces are glued together is 3 cm, and the slant height of the cone is 5 cm, work out the total volume and surface area of the spinning top (top A).

(2) If each top is packed in a box measuring 6.5 cm by 6.5 cm by 7.5 cm how many boxes will fit into a packing case with internal measurements 55 cm by 55 cm?

(3) You investigate other designs for two more tops, made the same way with the circular surface where the pieces are glued together having radius 3 cm. For top B the volume of the hemisphere is equal to the volume of the cone and for top C the surface area of the hemisphere is the same as the surface area of the cone. Find h cm, the height of the cone, for top B and for top C.

HELP

Find h from the right-angled triangle where r=3 and s=5.



The cone in the picture was made from the sector of the circle shown using scrap plastic.

Make your own cone and cylinder the same height using the templates on page 3, from paper or scrap card or plastic. Fill the cone with lentils, rice or something similar and investigate the relationship between the volume of the cone and the volume of the cylinder. This is not a proof of the formula but you will need to learn integral calculus to prove the formula.

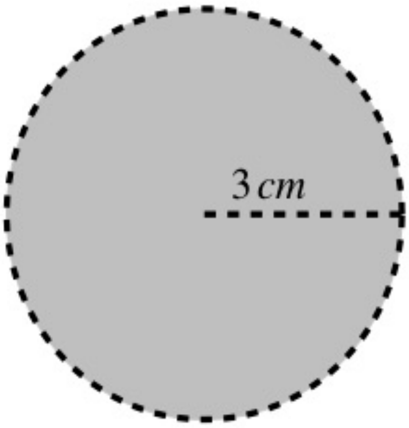
The formula for the volume of the cone is $\frac{1}{3}\pi r^2 h$ and the formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.

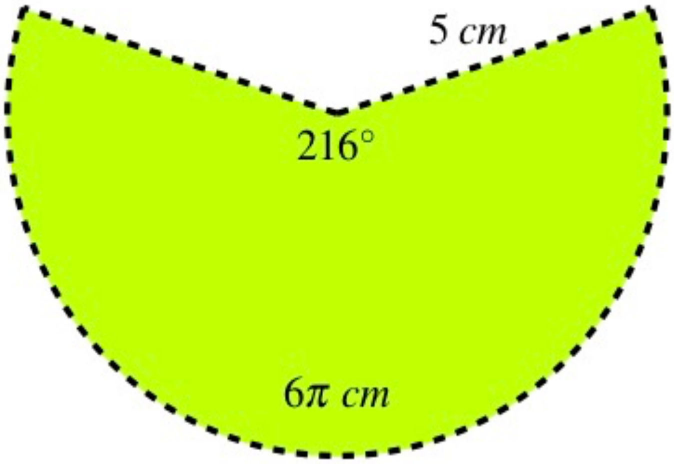
NEXT

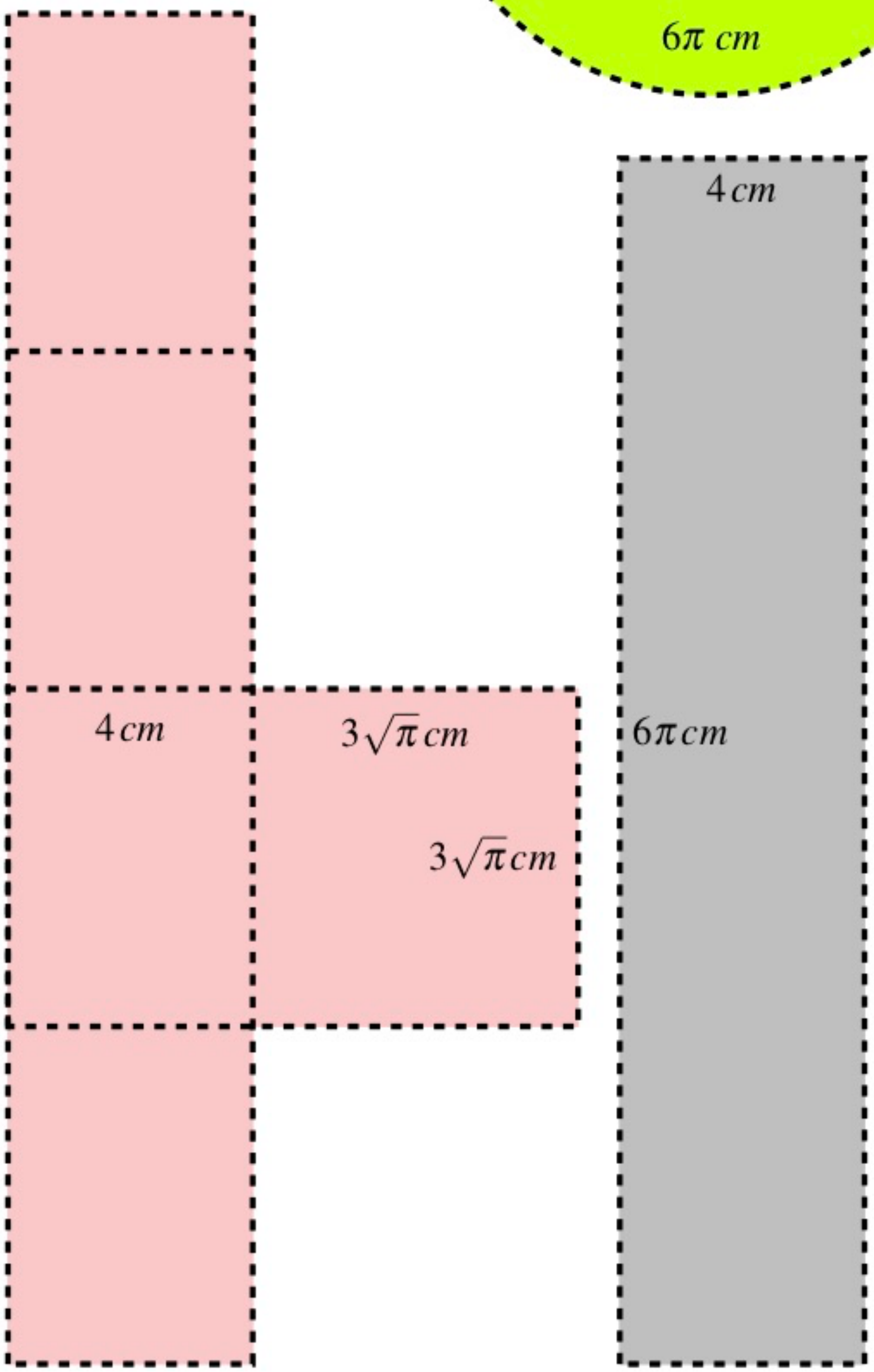
1. Work out the dimensions of a cuboid with the same volume as the cylinder and with the same height as the cylinder and cone.
2. The templates on page 3 can be used to make the three solids, the cone, cylinder and cuboid, which all have the same base area and height. The outlines can be marked on scrap card or scrap plastic by pricking through the template, or by tracing, or you could draw your own outlines using compasses and a protractor.
3. In another design the base radius r of top D is 5 cm. Can you find the height of the cone for which the total surface area of top D is numerically equal to the total volume.

Resources: Scrap plastic, sticky tape, scissors. Templates – see page 3)

TEMPLATES for making a cone, cylinder and cuboid
Scale 0.875:1







NOTES FOR TEACHERS

SOLUTION

(1) Given that $r = 3$ cm and $s = 5$ cm then $h = 4$ cm as this is a 3-4-5 triangle.
Total volume = volume of hemisphere + volume of cone
$= \mathbf{\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h}$
$= \frac{2}{3}\pi \times 27 + \frac{1}{3}\pi\times 9\times 4$
$= 18\pi + 12\pi = 30\pi \text{ cm}^3 = 94.25 \text{ cm}^3$ (to 2 decimal places)
Surface area = surface area of hemisphere + surface area of cone
$= \mathbf{2\pi r^2 + \pi rs}$
$= 18\pi + 15\pi = 33\pi \text{ cm}^2 = 103.67 \text{ cm}^2$ (to 2 decimal places).

(2) One line of 8 small boxes packed close together take up a space measuring 52 cm by 6.5 cm by 7.5 cm and so 64 small boxes can be packed in one layer in the packing case with a space of 1.5 cm all round.
Then, because $7 \times 7.5 = 52.5$ it is possible to pack 7 layers of 64 boxes in the packing case with 2.5 cm to spare.
<u>In this way 7× 64 = 448 boxes can be packed into the packing case.</u>
Note: It is wrong to divide the volume inside the packing case by the volume of one small box to get this answer because we have to allow for fitting the small boxes into the packing case.

(3) If the height of the cone is h then the slant height is $\sqrt{(h^2 + 9)}$ by Pythagoras Theorem.
Top B: If the volumes are the same then
$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$
$18\pi = 3\pi h$
<u>So $h = 6$ cm</u>

Top C: If the surface areas are the same then
$2\pi r^2 = \pi rs$
$18\pi = 3\pi\sqrt{(h^2 + 9)}$
$36 = h^2 + 9$
$h^2 = 27$
<u>$h = 3\sqrt{3}$ cm</u>

Why do this activity?

This activity is a little more challenging than simply working out volumes and surface areas by substituting in formulae. It requires learners to think about the composite shapes, to visualise how they can pack small cuboid shapes boxes into a packing case and, finally, to use the formulae to make equations satisfying the given conditions and to solve these equations to find the height.

Learning objectives

In doing this activity students will have an opportunity to:

- practise calculating the surface areas and volumes of spheres and cones and composite solids made up from them;
- practise applying knowledge of measures and volumes to real life packaging problems.

Generic competences

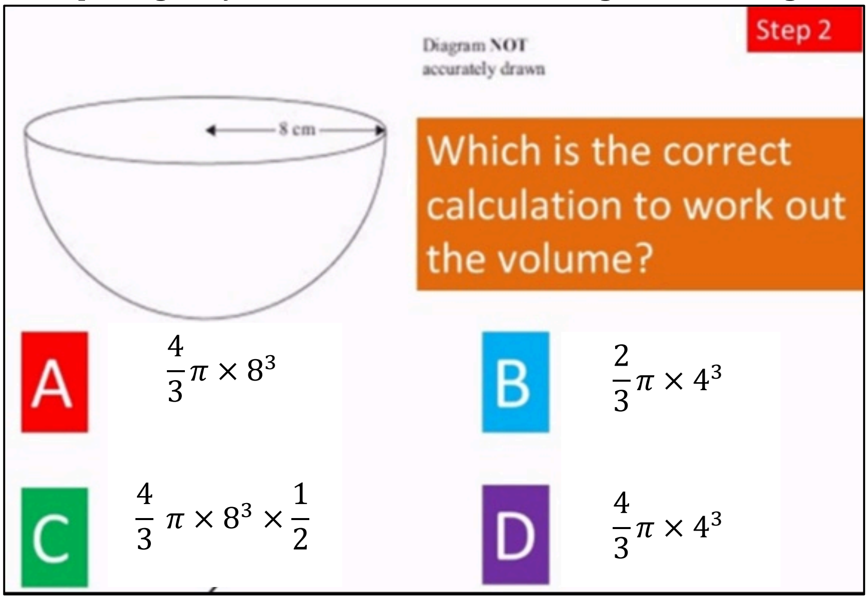
In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills to solve a problem;
- develop the **skill of visualizing and** interpreting visual images to represent concepts and situations.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.



1.Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2.It is important for learners to explain the reasons for their answers. Putting thoughts into words may improve communication skills understanding.
3.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. Ask the class to vote again or the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. The concept is needed for the lesson to follow, explain the right answer or give a remedial task.

C. is the correct answer.

Common Misconceptions

A. Forgotten to take half for the hemisphere.

B. Knew that for half a sphere you would use 2/3 to find the volume but mistakenly also halved the radius.

D. Halved the radius rather than halved the volume of the sphere.

https://diagnosticquestions.com

Suggestions for teaching

Start with the diagnostic question so that learners can confidently work out the volume of the top. Then give this exercise to the class in written form so that they get practice in reading and interpreting the question for themselves.

Use the 1 - 2 - 4 more teaching strategy. For about 20 minutes learners should work individually as if they are in an examination situation. Then ask the learners to consult each other **in pairs** to check answers and complete the questions for another 15 minutes. Then get each pair to consult with another pair, again to check answers and the methods used. Finally conduct a plenary where learners from different groups present an account of their working to the rest of the class explaining each step.

Key questions

- How are you going to find h if you know s and r?
- How would you pack the small boxes into the packing case?
- How many small boxes in each layer? How many layers?
- How can you use what you know to write down an equation which you can solve to find h?

Follow up

1. Find the dimensions of the cuboid with height 4 cm and volume equal to the volume of the cylinder of base radius 3 cm and height 4 cm.
2. Design small boxes for tops B, C and D and packing cases into which the smaller boxes can be packed.

MATHS TOYS

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13