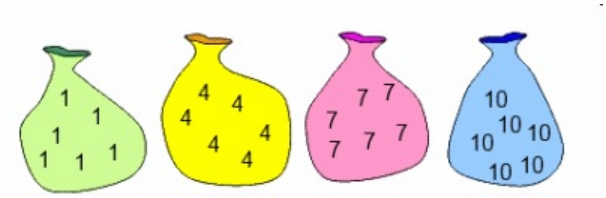
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

SPECIAL SUMS



Imagine you have four bags containing a large number of 1s, 4s, 7s and 10s.
You can choose numbers from the bags and add them to make different totals. You don't have to use numbers from every bag, and there will always be as many of each number as you need.

Choose some sets of 3 numbers and add them together.
What is special about your answers?
Can you explain what you've noticed?
Why does it happen?

HELP

Start with the easier and simple problem exploring what happens when you add two numbers chosen from bags containing 2s, 4s, 6s and 8s. What happens when you choose 3 numbers from the bags? Can you explain your findings?

NEXT

Work in pairs on the task. Explore what happens if you select four numbers from the bags. Then five, or six, or... Your big challenge will be to discover a rule that determines what is special if you select n numbers from the set of bags for any value of n.

Be prepared to explain to the class what would be special about the total if you added 99 numbers chosen from the bags, then to give the explanation for choosing n numbers. Make sure you can explain your reasoning with reference to the structure of the problem rather than just by spotting a pattern.

NOTES FOR TEACHERS

SOLUTION

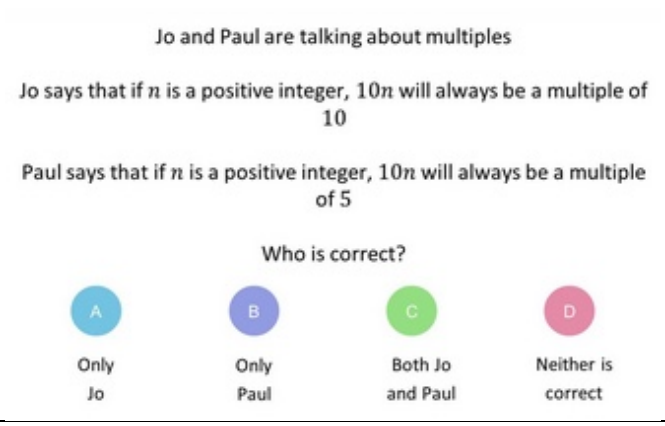
All the sums are multiples of three.
This is because all the numbers are one more than a multiple of 3 (a multiple of 3 plus 1). So when you add any three of these numbers you get a multiple of 3 plus 3 which is another multiple of 3.

Algebraic proof
The numbers are all of the form 3n+1 where n is a non negative integer.

Adding 3 of the numbers we get $(3n_1+1) + (3n_2+1) + (3n_3+1) = 3(n_1+n_2+n_3) + 3$ which is a multiple of 3.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is C

https://diagnosticquestions.com

Why do this activity?

This activity gives learners a practical context in which to think about the underlying structure behind multiples and remainders, as well as leading to some very nice generalisations and justifications.

Learning objectives

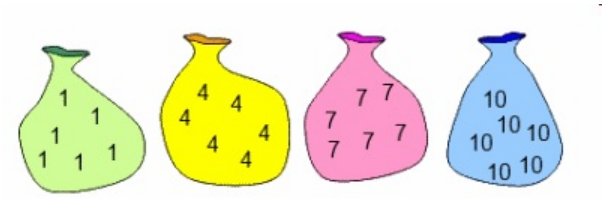
In doing this activity students will have an opportunity to:
practice addition and spotting number patterns;
develop number sense and understanding of multiples and remainders.

Generic competences

In doing this activity students will have an opportunity to develop of skills of making and justifying conjectures.

Suggestions for teaching

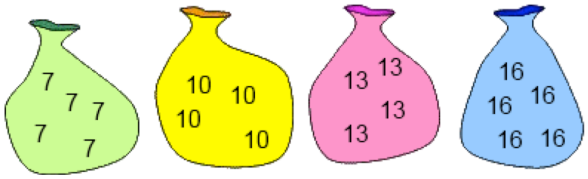
Start with the diagnostic quiz and emphasise that 10*n* just means 10 × some whole number *n*.



Display the image of the four bags. "Here are some bags containing lots of 1s, 4s, 7s and 10s. I'd like you to choose any three numbers from the bags and add them together."

Clarify that they can use numbers more than once if they like, then collect some of their answers on the board.

"What do you notice?" "Why are all these sums special?"
"What else do you notice?" *"They're all multiples of three".*
"Shall we make a conjecture that this always happens?"
"Try to find three numbers from the bags **that don't** add up to a multiple of three. If you can't do it, see if you can come up with an explanation of why it's not possible."



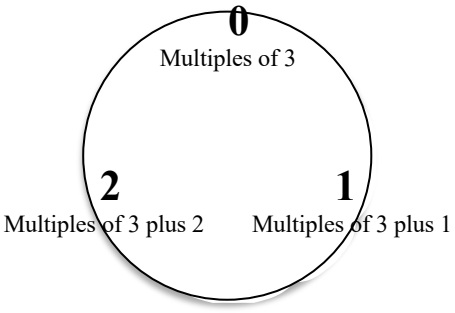
You could also use this image with 7s, 10s, 13s and 16s or suggest it to learners who are ahead of the others.

Give learners some time to work in pairs. While they are working, circulate and listen to their ideas. Then bring the class together to share insights.

Encourage learners to justify the conjecture. For younger learners this can be in words like

"all the numbers are 1 more then a number in the 3 times table. So if we add three of them it will give 3 more than a number in the three times table which is another number in the three times table."

For older learners you will want to encourage the use of algebra.



You might like to explain this in terms of clock arithmetic. Think of days of the week when, after 7 days, you get to the same day of the week again.
In this problem you see that all integers belong to one of 3 sets

- the set of multiples of 3,
- the set of 'multiples of 3 plus 1'
- and the set of 'multiples of 3 plus 2'.

Adding numbers is like doing arithmetic on a 3 hour clock counting round and round and round the clock.

Key questions

What's the smallest number I can make?
What's the next smallest?
What is special about the numbers in the bags?

Follow up

Elevenses https://aiminghigh.aimssec.ac.za/years-7-9-elevenses/ is about 11-hour clock-arithmetic.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6