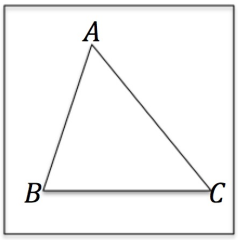
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

ALWAYS TRUE OR SOMETIMES TRUE?



Angles A, B and C are the angles of a triangle. Decide whether each of the following is an **identity**, always true for all triangles, or an **equation**, sometimes true. If you decide it is an equation find the solution or solutions and describe the corresponding triangle.

$\sin(180^o - A) = \sin B$

$\cos A = -\cos(B + C)$

$\cos A = \sin(B + C)$

Help

Have another go at this! Remember that the angles A, B and C are angles in a triangle so they add up to 180^o. Think about the graphs of the functions.

You most certainly can succeed in mathematics because the latest brain research proves that everyone can succeed if they believe in themselves and work hard. The causes for students to fail are mainly lack of self confidence together with lack of determination and lack of willingness to persevere.

Extension

Is $\tan(A + B) = -\tan C$ an equation or an identity?

What about $\sin(90^o – A) = \cos A$?

NOTES FOR TEACHERS

SOLUTION

1. $\sin(180^o - A) = \sin B$

From the sine function and its graph we know $\sin(180^o – A) = \sin A$
so $\sin(180^o - A) = \sin B$ is true if and only if $\sin A = \sin B$, that is only when $B = A$ or $180^o - A$.
This is an **equation** and $A = B$ is the only solution possible for a triangle. It is true for isosceles triangles where $A = B$.

2. $\cos A = - \cos(B+C)$

$\cos A = \cos(180^o – (B + C))$ as A, B and C are angles of a triangle and their sum is 180^o
$= - \cos(B + C)$ by the reduction formula for $\cos(180^o – \theta)$ for $\theta = A + B$
So this is an **identity** true for all triangles.

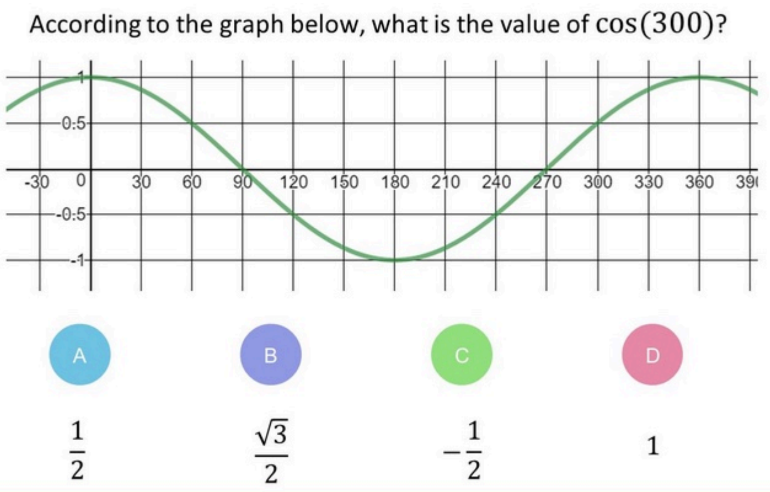
3. $\cos A = \sin(B + C)$

$\sin(B + C) = \sin(180^o – A)$ as A, B and C are angles of a triangle and their sum is 180^o
$= \sin A$ by the reduction formula
So $\cos A = \sin(B + C)$ is an equation and only true for triangles in which $\cos A = \sin A$, that is $A=45^o$ and angles B and C can have any value provided their sum is 135^o.

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.



5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is A :

B. This is the cosine of 30^o and also of 330^o.

C. This is the cosine of 120^o and 240^o.

D. This is the cosine of 0^o and 360^o
https://diagnosticquestions.com

Why do this activity?

This is a non standard task in which learners have to use what they know about the angles of a triangle and the addition formulae for sine and cosine and think mathematically about whether the statement is always true (an identity) or sometimes true (an equation). This is an important distinction and one that needs to be understood by learners.

Intended learning outcomes

In doing this activity students will have an opportunity to:
practice at simplifying trigonometric expressions, using reduction formulae, and solving simple trigonometric equations.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- think mathematically and to reason logically;
- analyze, reason and record ideas effectively.

Possible approach

Use the one-two-four-more teaching method. First get the learners to read the question for themselves and to work **individually** on it. Go around helping learners to get started if they are having trouble by asking questions but not telling them what to do. When you think that most learners have finished at least the first two examples get them to work in **pairs**, to compare their answers and to decide who is right if they disagree, then to finish the three questions together. It is important that they **both understand** and it helping a partner who is having difficulties gives benefits the other learner because it helps him or her to clarify their own thinking and to develop their communication skills.

When one pair has finished get the learners to work in **fours**, to compare their answers and to decide who is right if they disagree, then to finish the three questions together. As before, it is important that **all four learners understand** and helping each other benefits all the learners. Make it clear that you will choose learners to represent their groups to present their work to the class and not ask for volunteers. Groups who finish ahead of the class can be given the extension questions. It may be convenient to give the class these questions to finish for homework.
To complete this work, conduct a class discussion involving **all learners**. Ask learners to vote whether they think example 1 is an equation or an identity, everyone must commit themselves. Then call on a pair of learners to present their solution to the class. It works well for one learner to write the solution on the blackboard while the other learner explains it. How many learners voted for it to be an equation and how many had to change their minds. Repeat this for the other two parts.

In a one-two-four-more lesson slower learners get help from other learners. It is beneficial to take this into account when formulating a seating plan. Even with a very big class it is almost always possible for pairs of learners to turn around to work in a four with the pair behind them so that at least one of the four is a more confident and higher achieving student.

Key questions

- Can you use what you know about the angles of a triangle?
- What do you know about the sines of supplementary angles, that is angles x and 180 – x?
- What do you know about the cosines of supplementary angles, that is angle x and 180 – x?
- Is that true for all values of A, B and C or just for some values?

Follow up

Tangled Trig Graphs https://aiminghigh.aimssec.ac.za/years-10-12-tangled-trig-graphs/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6