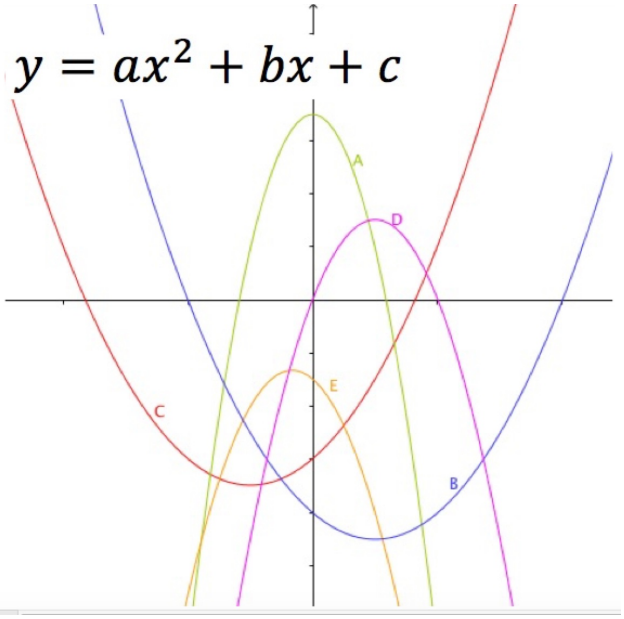
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

QUADRATIC FUNCTIONS

The diagram shows the graphs of five quadratic functions or parabolas with equation $y = ax^2 + bx + c$ for different values of a, b and c.



How do the two points where the graph cuts the x-axis relate to the axis of symmetry of the graph and the solutions of the quadratic equation $ax^2 + bx + c = 0$?

Match the graphs in the diagram to the following descriptions and give reasons for your decisions.

1. $y = ax^2 + bx + c$ if $a > 0, b > 0$ and $c < 0$
2. $y = ax^2 + bx + c$ if $a < 0, b = 0$ and $c > 0$
3. $y = ax^2 + bx + c$ if $a < 0, b < 0$, and $b^2 - 4ac < 0$
4. $y = a(x + p)^2 + q$ if $p < 0, q < 0$ and the x-intercepts have different signs.
5. $y = a(x + p)^2 + q$ if $a < 0, p < 0, q > 0$ and one root is zero

HELP

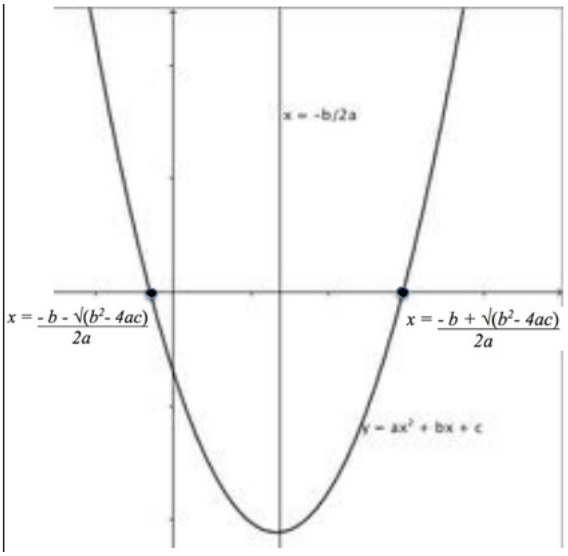
For each case use the descriptions and choose values of a, b, c, p and q that fit the descriptions. You might try sketching the graph with your choice of these values. This should help you to match it to one of the coloured graphs A, B, C, D & E.

NEXT

Can you re-produce exactly the same diagram for the coloured graphs A, B, C, D & E and find the equations of the graphs. Perhaps use Geogebra. You can download the software for free from https://www.geogebra.org/ and use it on your smartphone without being connected to the internet.

NOTES FOR TEACHERS

SOLUTION



The graph of the quadratic function $y = ax^2 + bx + c$ is a parabola with axis of symmetry $x = -b/2a$.

This is because the solutions of the equation are:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

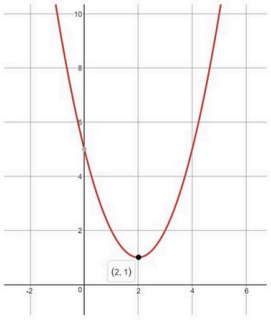
The solutions of the quadratic equation $y=ax^2 + bx + c = 0$ correspond to the points where the graph of $y = ax^2 + bx + c$ cuts the x-axis. These points $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ are symmetrically placed on either side of the line $x = -b/2a$ as shown in the diagram.

Graph C matches description 1.
Graph A matches description 2.
Graph E matches description 3.
Graph B matches description 4 because $y = a(x + p)^2 + q$ is the graph of $y = x^2$ stretched in the y direction by a factor a, and translated p units in the positive x-direction and q units in the y direction.
Graph D matches description 5.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Which of the following could be the equation of this curve?



A $(x-2)^2 - 1$

B $(x+1)^2 - 2$

C $(x-2)^2 + 1$

D $(x+2)^2 + 1$

2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is C. This can be easily checked by substituting $x = 2$ in the equation.
If learners give any other answers it shows they have made a guess or used what they have learned without understanding. https://diagnosticquestions.com

Why do this activity?

By sketching graphs of quadratic functions without knowing the exact values for the variables learners have to draw on their knowledge of the symmetric properties of the graphs and of transformations of graphs.

Learning objectives

In doing this activity students will have an opportunity to:

- make connections between solving algebraic equations and their graphs;
- appreciate how and why the effect of changing the parameters in equations of functions is connected to finding the solutions of equations;
- gain a deeper understanding of quadratic functions;
- generalise their knowledge of graphs of quadratic functions by drawing sketch graphs without knowing the exact values for the variables.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for teaching

The teacher could give the list of 5 descriptions to the class and ask them to sketch the graphs, then have a class discussion where the learners are asked to draw the sketch graphs on the board and everyone has to agree that the graph drawn matches the description. Then the learners could copy the correct sketch graphs in their notebooks.

Another possible approach, to save time, is to give the 5 descriptions to different pairs of learners so that several pairs of learners have to sketch each graph but initially everyone does not have to sketch all the graphs.

Another approach is to give the list of 5 descriptions to the class and ask them to sketch the graphs, then after most of the learners have done this, to give out the diagram showing the coloured graphs A, B, C, D & E and ask the learners to match the descriptions to the graphs in the diagram.

Alternatively learners could experiment by drawing graphs using Geogebra or some other graphing software.

Key questions

- Which way up is the graph? How is this connected to whether a is positive or negative?
- Does the graph have a maximum point or a minimum point?
- Are the coordinates of the maximum (or minimum) point on the graph positive or negative? What does that tell you?
- Where does the graph cut the x-axis? What does that tell you about the solutions of the corresponding quadratic equation?
- How does that graph relate to the graph of $y = x^2$? What does that tell you about the equation of the graph?

Follow up

Graphing Quadratic Equations
https://aiminghigh.aimssec.ac.za/years-10-12-graphing-quadratic-equations/

Quadratic Equations https://aiminghigh.aimssec.ac.za/years-10-to-12-quadratic-equations/
Quadratic Matching 1 https://aiminghigh.aimssec.ac.za/grades-10-to-12-quadratic-matching-1/
Quadratic Matching 2 https://aiminghigh.aimssec.ac.za/grades-10-to-12-quadratic-matching-2/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see https://nrich.maths.org/12339 Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6