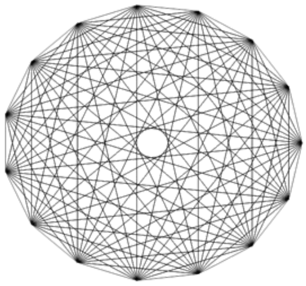
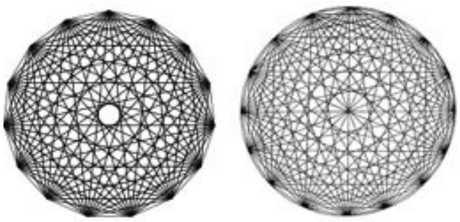
MYSTIC ROSE

What can you see in this diagram?



Could you draw it? Could you draw a similar diagram with just 5 or 6 points around the outside? Try it?

How many lines are there in your diagrams? How many lines are there in the original diagram? Can you find a way of working out the number of lines without counting them? Can you explain your method?



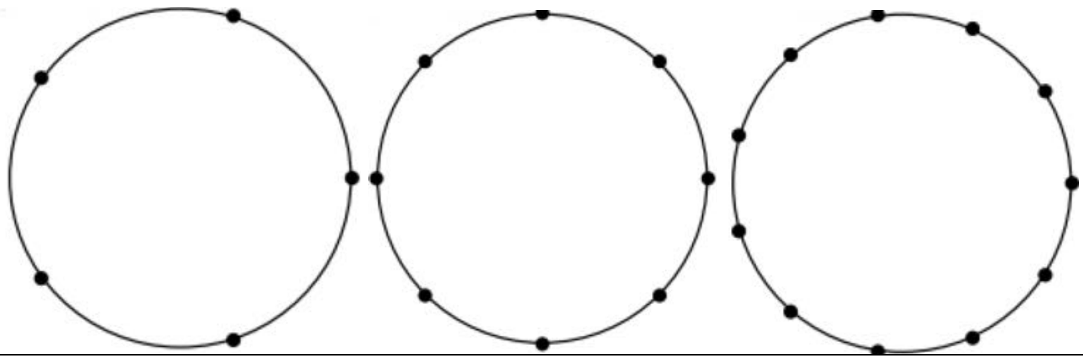
Are these two diagrams the same or different?

Explain your answer?

See also the NRICH Mystic Rose Poster

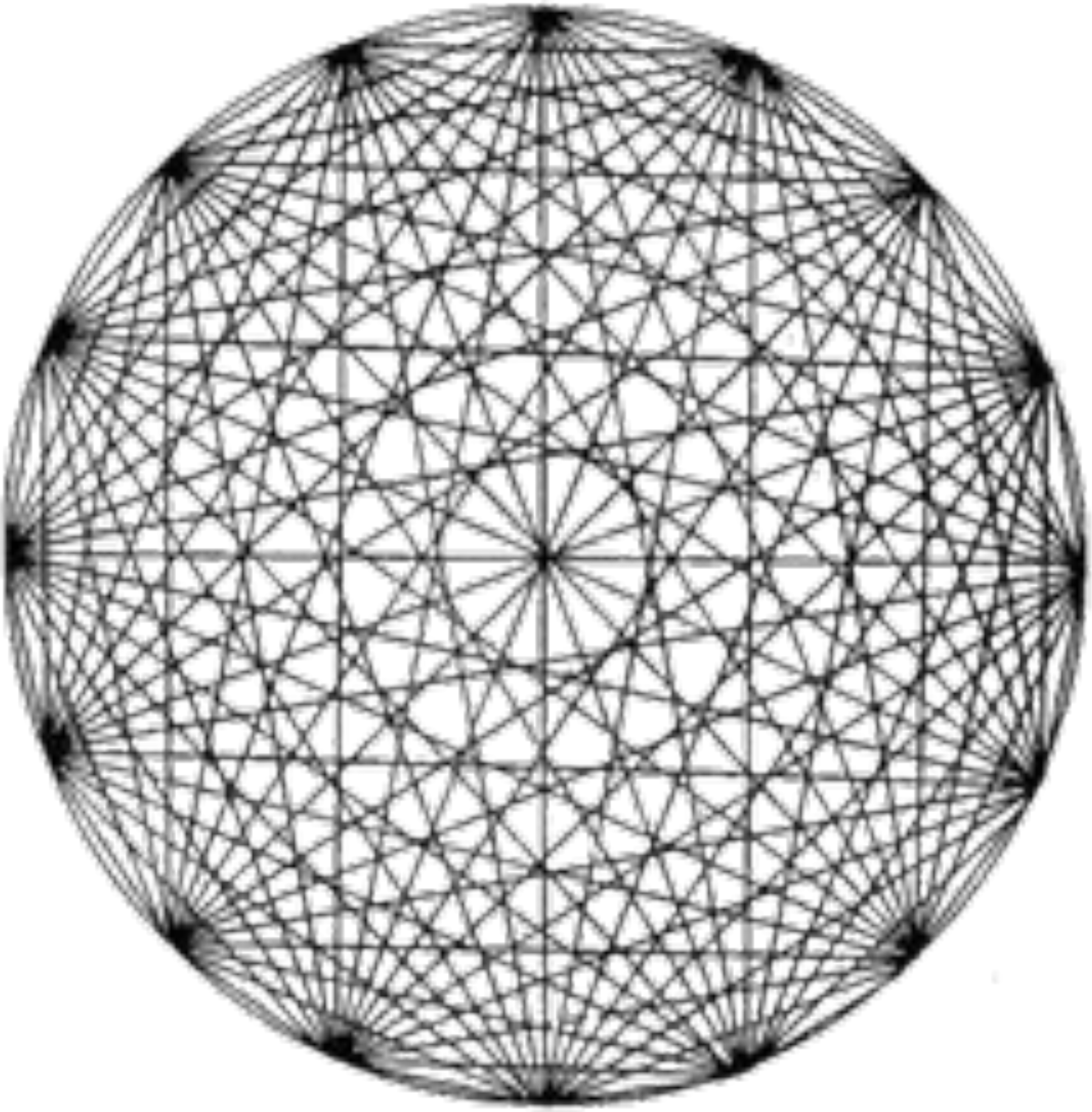
HELP

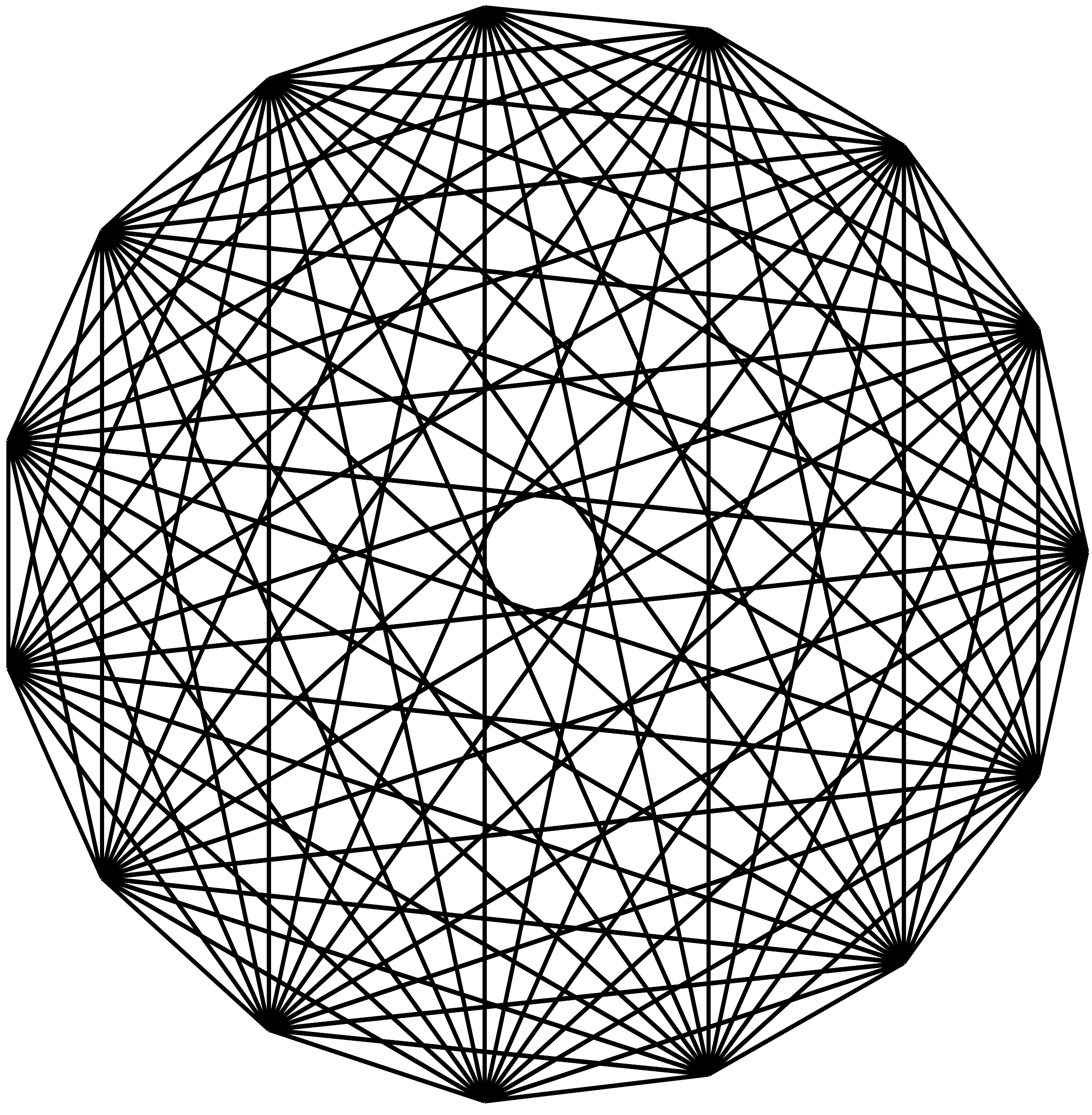
Draw 3 mystic roses by joining all the dots on these circles to all the other dots by straight lines and answer the questions above.



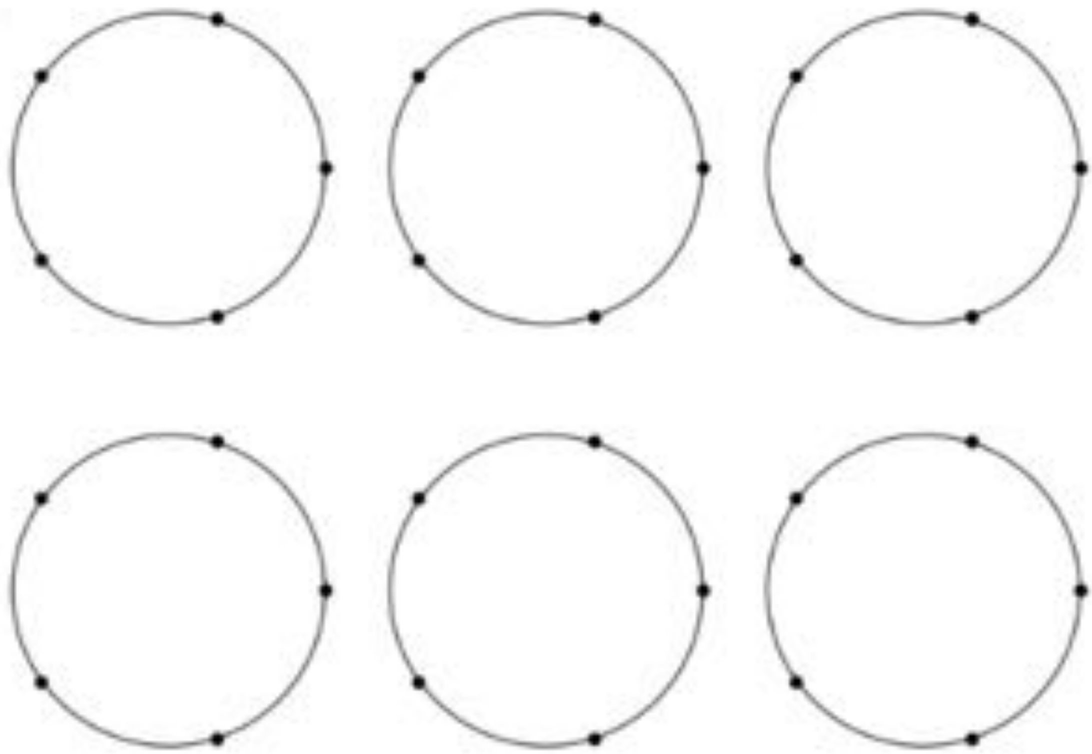
NEXT

Suppose you are one of 12 people at a meeting and everyone shakes hands with everyone else. How many handshakes would there be? Explain how you found your answer.

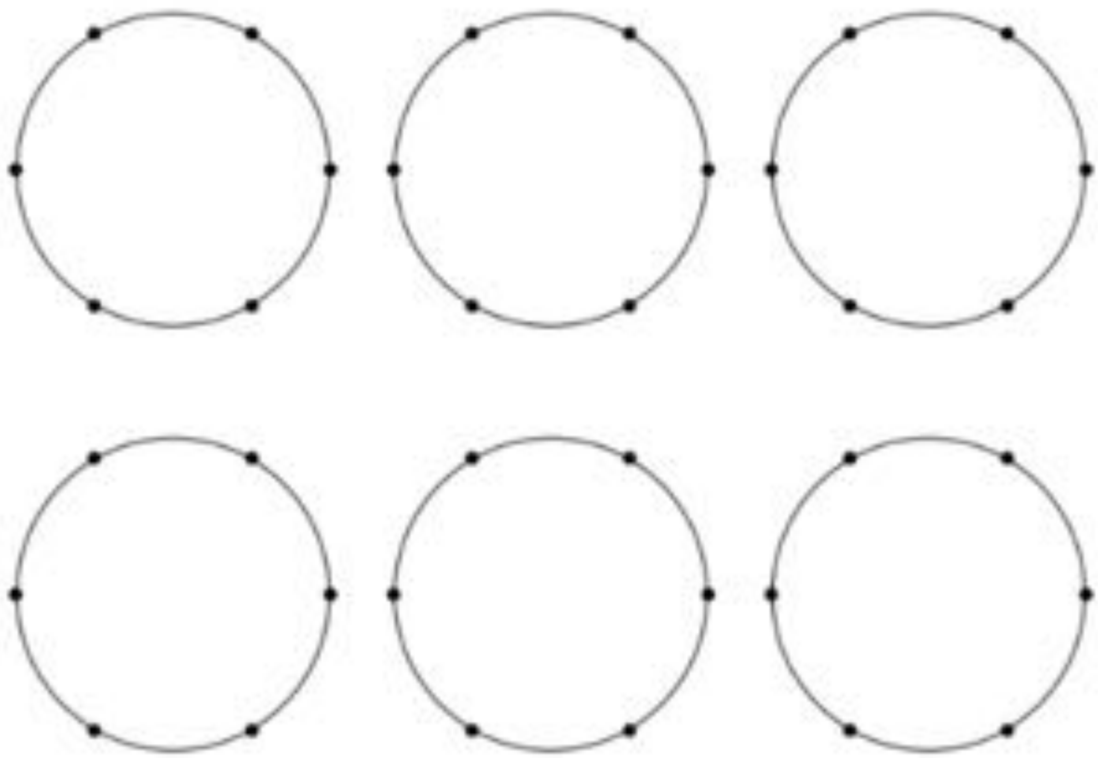




5 point circles



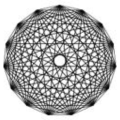
6 point circles

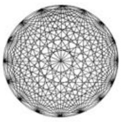


NOTES FOR TEACHERS

SOLUTION

If there are n points around a circle and every point is joined with a straight line to





every other point there will be $\sum_{i=1}^{n-1} i = \frac{n(n-1)}{2}$ lines. The diagram is called a Mystic Rose.

If the number of points is even there will be lines through the centre of the circle and the diagram will be symmetrical about these diameters.

If the number of points is odd there will be an empty circular space around the centre of the circle. **The chords form the envelope of this circle.** The lines of symmetry in the diagram are diameters joining the points on the circle to points midway between the two on the opposite side.

Method 1

For 6 points we join the 1st point to the other 5 points with 5 lines. To join the 2nd point to all the other points we need 4 lines as it is already joined to the 1st point. Similarly we need 3 more lines to join the 3rd point to the remaining points and 2 lines for the 4th point, and 1 line for the 5th point.

So there are 5 + 4 + 3 + 2 + 1 = 15 lines.

For n points there are 1 + 2 + 3 + … + (n-1) lines.

Method 2

Some people may work this out that for each of the 6 points on the circle there 5 points at the other end of the chord drawn from that point giving 6 x 5 = 30.

But, counting the chords, this is too many because you are counting every line twice so you need $\frac{1}{2}$(6 x 5) lines, that is 15 lines.

For n points there are $\frac{1}{2}n$ (n-1) lines.

The two methods together give extra information:
the number of chords for n points on the circumference of a mystic rose;
the sum of the natural numbers 1 + 2 + 3 + … + (n – 1)

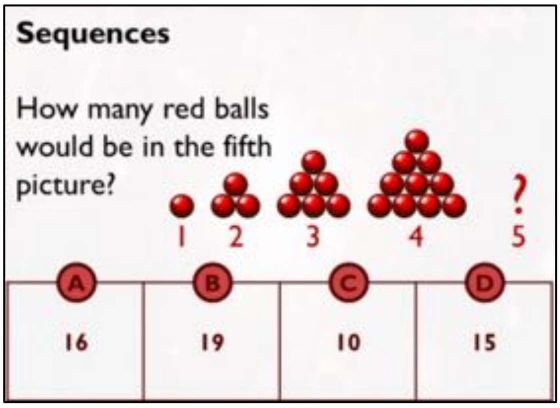
1 + 2 + 3 + ... + (n-1) = $\frac{1}{2}n$ (n – 1)

In the diagram there are 15 points and (15 x 14)/2 = 105 lines.

The two diagrams differ in that the first has an odd number of points so none of the lines go through the centre of the circle whereas the second has an even number of points and all the lines go through the centre of the circle.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. It is important for learners to explain the reason for their answers. Putting their thoughts into words helps them to think more clearly and to develop communication skills.
4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.



5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
6. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is D.
A., B. and C. Learners giving these answers have not been able to imagine adding a line of 5 red balls to the 4th triangular pattern. https://diagnosticquestions.com

Why do this activity?

This activity is one of a set of three related activities (Handshakes, Triangle Number Picture and Mystic Rose) where **the same mathematics and the same formula** occurs in **three different contexts.** It is important for learners to appreciate the connection. The mathematical term to describe this is ***'isomorphism'***, that is different mathematical examples that have the same mathematical structure. This is not a term that learners need to know but they should meet such examples and be prompted to ask *"Have we seen anything like this before?"*

Teachers may use the Handshakes example in primary school without any algebra as this is a practical activity that learners can understand at this stage and easily find answers for small groups and generalise this to larger groups. Later teachers might plan the progression to algebra, introducing learners to the other two activities and making the connections between all three, using algebra as appropriate to the class.

Learning objectives

At the end of this activity learners will:

- ✓ know how to find the sum of the first n natural numbers;
- ✓ understand how the formula for the sum can be derived;
- ✓ be able to make conjectures and generalisations;
- ✓ appreciate the underlying structure of a pattern or problem;
- ✓ if they also meet Handshakes or Triangle Numbers they will experience multiple representations of the same pattern.

Generic competences

In doing this activity students will have an opportunity to:

- ✓ **think mathematically,** reason logically and give explanations and proofs;
- ✓ **think flexibly**, be creative and innovative;
- ✓ develop the skill of interpreting and **creating visual images** to represent concepts and situations;
- ✓ interpret and **solve problems** in a variety of situations;
- ✓ **communicate** in writing, speaking and listening:
- ✓ exchange ideas, criticise, and present information and ideas to others and record ideas effectively.

Curriculum content:

School Years/Grades 4 – 6

Observe and describe relationships or rules in the learner's own words
Investigate and extend numeric and geometric patterns looking for relationships or rules of patterns.
Solve problems involving whole numbers and number patterns.

School Years/Grades 7 – 9

Investigate and extend numeric and geometric patterns looking for relationships between numbers.
Describe and justify the general rules for observed relationships between numbers in their own words or in algebraic language.

School Years/Grades 10 – 12

Investigate number patterns leading to those where there is constant difference between consecutive terms and the general term is therefore linear.
Sum of an arithmetic series.

Suggestions for Teaching

Resources Mystic Rose poster. Click here for a template of circles with 5, 6 and 24 dots for drawing lines and investigating these patterns (see page 4).

Put the mystic rose poster where everyone can see it if you have a large poster or copy the diagrams on pages 2 and 3.

Place a chair a few feet in front of the poster. Learners take turns to sit in the seat and make a short comment about the poster. Only the person in the 'hot seat' in front of the poster can speak. The comment can be general or mathematical.

Accept all comments. Some may be about the circles people see or about liking the pattern. There is often an argument about whether the poster is made up of circles or straight lines or perhaps not really circles but polygons with many sides. Eventually people will realise that the shape is entirely made up of straight lines.

***After a while ask the learners how they think the poster was drawn – do they think they could draw the poster.**

Ask the learners to draw a similar shape with just 5 or 6 points round the outside.

How many lines will there be in their diagrams?

If possible have the learners work in groups to answer this question. In most classes some groups will find the answer using Method 1 and other groups will use Method 2. **It is very important that the teacher does NOT try to guide them to use one method or the other.**

When most groups have found a solution then lead a class discussion where the learners explain their methods. The class should discuss the fact that the answers will be the same so that, with more points, they can avoid adding up all the terms of the sequence and instead use a formula.

Then ask

**How would you answer the question for more points on the circle?*

**How many lines are there in the original diagram?*

** How many lines would be in a pattern with 100 points?*

Then for learners in secondary school, ask for a generalisation:

**Can they give a general method for finding the answer? Can they write this as a formula?*

Make the connections with triangle numbers.

Key questions

- How many points are there around the circle?
- Can you count the number of straight lines in the diagram?
- Suppose you only have 3 points on the circle and every point is joined to every other point, than how many lines would there be?
- What about the number of lines for 4 points, or 5 points, or 6 points …
- If you had 100 points you would not want to count the lines but could you work out how many lines there would be?

Follow up

Handshakes https://aiminghigh.aimssec.ac.za/handshakes/
Triangle Number Picture https://aiminghigh.aimssec.ac.za/triangle-number-picture/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12				
	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13