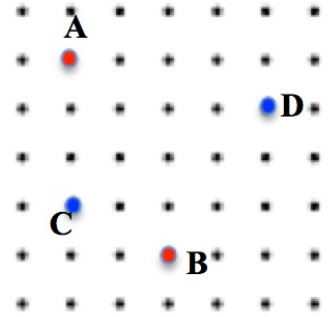
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

DOTTY RELATIONS

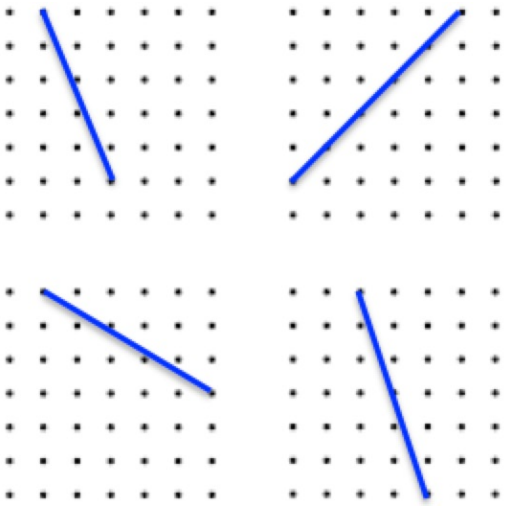


On the grid join the two red dots, A and B, together with a straight line.

Now join the blue dots, C and D, with a straight line.

At what angle do the two lines cross?

Investigate the number of squares "along" and "down" from A to B compared with the number of squares "along" and "up" from C to D. What do you notice?

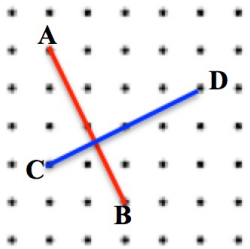


Using what you have discovered draw lines on these grids perpendicular (at right angles) to the lines shown.

Can you find more than one solution each time? Why or why not?

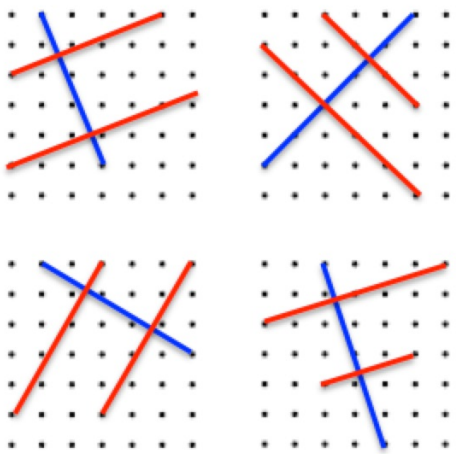
Does it matter how long the lines are? Why or why not?

SOLUTION



The lines cross at right angles.

The number of squares "along" from A to B is the same as the number of squares "up" from C to D that is 2 squares. The number of squares "down" from A to B is the same as the number of squares "across" from C to D that is 4 squares.



The red lines in the diagram are perpendicular to the blue lines and there are many other possible solutions. Any line parallel to the red line will be another solution.

The lines can be any length because it is the ratio of "distance up" to "distance across" that determines the direction of the line (and the gradient).

The ratios $\frac{\text{distance up}}{\text{distance across}}$ for perpendicular lines must have opposite signs and must be reciprocals.

Example $2/6 = 1/3$ and $-6/2 = -3$

NOTES FOR TEACHERS

Why do this activity?

This activity could be linked to coordinates. It could also form an introduction to vectors at a higher level. It is a powerful geometrical investigation. It has the potential to lead quickly to generalisations that learners can apply in the second part of the question.

Intended learning outcomes – Grades 7 and 8

- To develop an appreciation for the use of constructions to investigate the properties of geometric figures, in particular perpendicular lines and parallel lines.
- To develop a clear and precise description of the condition for 2 lines to be perpendicular

Possible approach

Either print the top box from page 1 and give out copies to the learners so that they can read the question and draw the required lines on the sheet or write the question on the board and give out dotty paper so that the learners can draw the diagrams on it.

When most of the learners have completed the activity lead a class discussion in which the learners explain what they have discovered. The teacher should ask the learners to work out the fraction

$$\frac{\textbf{distance up}}{\textbf{distance across}}$$

for any two points chosen on the line and then help the learners to understand that this ratio will be the same whichever two points are chosen.

You can tall the class that this ratio is called ***the gradient of the line***.
The outcome of this lesson should be that learners have discovered for themselves that if two lines are at right angles (perpendicular) then the product of the gradients is equal to -1.

Key questions

What is the "distance across" from this point to that? Is it positive or negative?
What is the "distance up" from this point to that? Is it positive or negative?
What is the ratio $\frac{\textbf{distance up}}{\textbf{distance across}}$ for two points on that line?
How are you deciding whether those two lines are perpendicular?

Possible extension

The ratio $\frac{\text{distance up}}{\text{distance across}}$ for two points on a line is called the gradient.

Learners could be given some gradients and asked to draw lines with those gradients.

The teacher could introduce coordinates and ask the same questions.

Possible support

Use the corner of a piece of paper or cardboard to check that angles are right angles.