

AIMING HIGH

QUADRATIC FUNCTIONS

If you have a smartphone, tablet or computer, then you can download Geogebra, which is very powerful FREE software. <u>https://www.geogebra.org/</u> It comes with many helpful guides for using the software and ideas for lessons on functions, geometry and statistics.

The diagram shows the graphs of five quadratic functions or parabolas with equation



How do the two points where the graph cuts the *x*-axis relate to the axis of symmetry of the graph and to the solutions of the quadratic equation $ax^2 + bx + c = 0$?

Match the graphs in the diagram to the following descriptions and give reasons for your decisions.

 $1.y = ax^{2} + bx + c \text{ if } a > 0, b > 0 \text{ and } c < 0$ $2.y = ax^{2} + bx + c \text{ if } a < 0, b = 0 \text{ and } c > 0$ $3.y = ax^{2} + bx + c \text{ if } a < 0, b < 0, \text{ and}$ $b^{2} - 4ac < 0$

 $4.y = a(x + p)^2 + q$ if p < 0, q < 0 and the *x*-intercepts have different signs.

5. $y = a(x + p)^2 + q$ if a < 0, p < 0, q > 0 and one root is zero.

HELP

For each case use the descriptions and choose values of a, b, c, p and q that fit the descriptions. You might try sketching the graph with your choice of these values and you could do this with Geogebra. This should help you to match it to one of the coloured graphs A, B, C, D & E.

NEXT

Can you re-produce exactly the same diagram for the coloured graphs A, B, C, D & E and find the equations of the graphs. Perhaps use Geogebra. You can download the software for free from <u>https://www.geogebra.org/</u> and use it on your smartphone without being connected to the internet.

GUIDE FOR PARENTS

Think about the future that lies ahead for your children. To do well in life, learners need to be confident that they can learn and apply new ideas. The quality of your children's lives now, and as adults, will be vastly improved if they have natural curiosity and enjoy learning, and if they like problem solving and finding out for themselves independently. They need a GROWTH MINDSET which means knowing, and having confidence, that they can succeed if they persevere and work hard. We need to prepare young people for a job market where existing knowledge and skills have limited value unless they can be applied in new ways to solve today's complex problems and to improve the quality of life for all. Share this idea with your children in an age appropriate way.



One of the most important and useful ideas in mathematics is the idea of a function.

The graph of the quadratic function $f(x) = y = ax^2 + bx + c$ is a parabola as shown in the diagram with axis of symmetry x = -b/2a.

The solutions of the quadratic equation

 $y=ax^2 + bx + c = 0$ correspond to the points where the graph of $y = ax^2 + bx + c$ cuts the *x*-axis. These points $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are symmetrically placed on either side of the line x = -b/2a as shown in the diagram.

Graph C matches description 1. Graph A matches description 2.

Graph E matches description 3.

Graph B matches description 4. The graph of $y = a(x + p)^2 + q$ is the graph of $y = x^2$ stretched in the y direction by a factor *a*, and translated p units in the positive *x*-direction and q units in the *y* direction.

Graph D matches description 5.

Why do this activity?

By sketching graphs of quadratic functions without knowing the exact values for the variables learners have to draw on their knowledge of the symmetric properties of the graphs and of transformations of graphs.

Learning objectives

In doing this activity students will have an opportunity to:

- make connections between solving algebraic equations and their graphs;
- appreciate how and why the effect of changing the parameters in equations of functions is connected to finding the solutions of equations;
- gain a deeper understanding of quadratic functions;
- generalise their knowledge of graphs of quadratic functions by drawing sketch graphs without knowing the exact values for the variables.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations;
- think flexibly, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Guide for homelearning

It's best to give this question to your learner with the list of 5 descriptions and suggest that they sketch possible graphs to fit the descriptions and after that to match the coloured graphs to the descriptions.

You could download the Geogebra free software and learners could use it to experiment by drawing graphs and seeing if they turn out as they expected.

One of the very best ways for people to learn is to explain something to someone else. One of the best ways that you can help your children with their school-work is to ask them to explain bits of it to you. You can do this here.

Ask questions if you don't understand. Perhaps the young person could give a better explanation if he or she thought a bit more about it. It also works well for younger siblings to test their older brothers and sisters on work they have to revise for a test. In this case most learners will have met functions in secondary school and younger children can ask older ones to tell them more about why functions are important.

Key questions

- Which way up is the graph? How is this connected to whether *a* is positive or negative?
- Does the graph have a maximum point or a minimum point?
- Are the coordinates of the maximum (or minimum) point on the graph positive or negative? What does that tell you?
- Where does the graph cut the *x*-axis? What does that tell you about the solutions of the corresponding quadratic equation?
- How does that graph relate to the graph of $y = x^2$? What does that tell you about the equation of the graph?



Follow up

Graphing Quadratic Equations

https://aiminghigh.aimssec.ac.za/years-10-12-graphing-quadratic-equations/ Quadratic Equations https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-equations/ Quadratic Matching 1 https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-1/ Quadratic Matching 2 https://aiminghigh.aimssec.ac.za/years-10-12-quadratic-matching-2/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the				
USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.				
For resources for teaching A level mathematics see <u>https://nrich.maths.org/12339</u>				
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.				
	Lower Primary	Upper Primary	Lower Secondary	Upper Secondary
	Age 5 to 9	Age 9 to 11	Age 11 to 14	Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6