EGYPTIAN FRACTIONS

Old manuscripts show that Egyptian fraction notation was developed around 2000 BCE. Egyptian wrote fractions as unit fractions, that is fractions with a numerator of 1. This was useful in sharing goods or land. For example to share 3 pizzas between 5 people you could give each person ½ a pizza and then share the remaining ½ between 5 giving each person 1/10th. This notation was used up until the Middle Ages. Fibonacci developed methods to write all rational numbers as sums of Egyptian fractions as described in his book Liber Abaci published in 1202.

We are going to investigate methods of reducing unit fractions to the sums of different smaller unit fractions as we see in this example:

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

Try some for yourself. Does this always work?

What do you notice about the fractions on the right in these expressions?

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

Can you use this idea to write down 1/5 as the sum of two smaller unit fractions?

$$\frac{1}{4} = \frac{1}{5} + \frac{1}{20} \dots$$

Can you explain to someone else how to use this method?

Perhaps we can find other ways to reduce unit fractions to the sums of different smaller unit fractions. For example:

$$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

Can all unit fractions be made up in more than one way like this? Why?

Try some examples of your own to test out your ideas.

$$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$$

$$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$$

$$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{6} = \frac{1}{11} + \frac{1}{14} + \frac{1}{231} \dots$$

HELP

1/6+1/7=13/42 😦

1/6+1/8=7/24 😦

…

1/6+1/12=1/4 🙂

Explore some unit fraction addition calculations like the example given. Highlight any that result in a unit fraction as an answer. Start with 1/3 which quickly gives you the smiley, then do the list for 1/4, then for 1/5. Sometimes you need to list many calculations before you get an answer that is a unit fraction.

NEXT

Can you prove either algebraically or visually that $\dfrac{1}{n} = \dfrac{1}{n+1} + \dfrac{1}{n(n+1)}$?

Can you predict how many different pairs of unit fractions will add up to any given unit fraction?

The Greedy Algorithm https://aiminghigh.aimssec.ac.za/the-greedy-algorithm/