AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

UNDOING

Amy and her friends have built some functions and they are challenging each other to find the input when they know an output.

Amy's function $a \rightarrow$ [+5] $\rightarrow a+5$

Busi's function $b \rightarrow$ [×3] $\rightarrow$ [−2] $\rightarrow 3b-2$

Chris's function $c \rightarrow$ [÷2] $\rightarrow$ [×3] $\rightarrow$ [+5] $\rightarrow 3c/2 + 5$

Dudu's function $d \rightarrow$ [-2] $\rightarrow$ [×3] $\rightarrow$ [÷2] $\rightarrow$ [+5] $\rightarrow 3(d-2)/2+5=3d/2+2$

They think the inputs to their functions that all give the output 10 are 5 for Amy's function, 4 for Busi's, 2½ for Chris's and 5⅓ for Dudu's. Do you agree? Why or why not?

Busi says that she goes back in the other direction to find inverses undoing the functions one by one. For her function, to find the input that gives the output 10 she works out 10+2=12 and 12÷3=4.

Amy says she uses inverse functions because they undo the operation of a function like undoing your shoelaces. She says that + and – undo each other and × and ÷ undo each other.

−2 **× 3** **÷2** **+5** What are the four inverse functions for these simple functions?

Can you find the inputs for Amy's, Busi's, Chris's and Dudu's functions corresponding to an output of 20?

With a partner decide on one of the functions, or build another function of your own. Give each other an output and challenge the other to find the input. Which of you can do this most quickly and accurately?

The formulas for the functions are given in the diagram.

HELP

The important idea to understand is that + and – are inverse operations and × and ÷ are inverse operations. Just like locking and unlocking a door these pairs of operations reverse the action of each other.

The inverse of Chris's function is $f^{-1}(x) = \frac{2}{3}(x-5)$.

Can you find the inverse of Amy's. Busi's and Dudu's functions?

NEXT

For each of the functions: f(x) = ⅜(x -7) and f(x)=⅚x + 2

1. Break each function up into simple steps.
2. Find the formula for the corresponding inverse function.
3. Find the input for the outputs 3, 12 and 27.
4. With a partner both think of an input and find its output for one of these functions, then exchange outputs and see who is first to find the input.

Notes for teachers

Solution

If the output for Chris's function is 10 then, to find the input, calculate 10 - 5 = 5, then 5 ÷ 3=1⅔ then 1⅔ × 2 = 3⅓ so the input should be 3⅓ and 4½ is wrong.
For Amy's function 5 -> 10, for Busi's 4->10 and for Dudu's 5⅓ -> 10.

The function x -> x -2, also written f(x) = x – 2 , has inverse $f^{-1}(x) = x + 2$.
The function x -> 3x, also written f(x) = 3x, has inverse $f^{-1}(x) = x/3$.
The function x -> x/2, also written f(x) = ½x has inverse $f^{-1}(x) = 2x$.

Amy's function a -> a + 5 or f(x) = x + 5 has inverse $f^{-1}(x) = x - 5$ so $f^{-1}(20)=15$

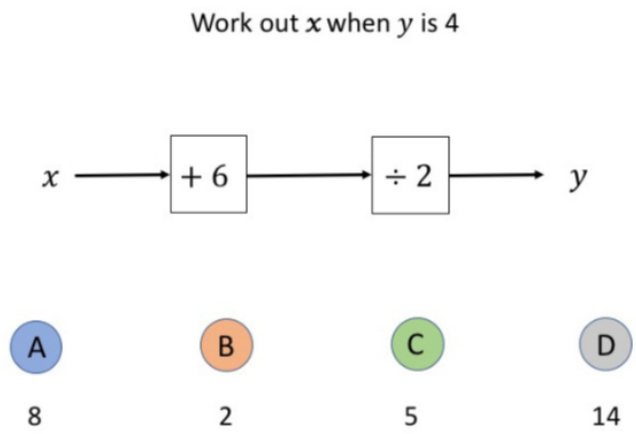
Busi's function b -> 3b – 2 or f(x) = 3x - 2 has inverse $f^{-1}(x) = ⅓(x + 2)$ so $f^{-1}(20)=22/3$.
Or calculate 20+2=22, 22÷3=22/3

Chris's function c -> 3c/2 + 5 or f(x) = 3x/2 + 5 has inverse $f^{-1}(x) = ⅔(x – 5)$ so $f^{-1}(20)=10$.
Or calculate 20-5=15, 15÷3=5, 5×2=10

Dudu's function d -> 3d/2 + 2 or f(x) = 3x/2 + 2 has inverse $f^{-1}(x) = ⅔(x – 2)$ so $f^{-1}(20)=12$
Or calculate 20-5=15, 15×2=30, 30÷3=10, 10+2=12

Diagnostic Assessment This should take about 5 minutes.

1. Write the question on the board and ask the class to **"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".**
2. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank them for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



B. is the correct answer. If we want to work out x we work backwards. We first do the inverse of division which is multiplication. 4 multiplied by 2 equals 8. Then we do the inverse of addition which is subtraction. 8 subtract 6 equals 2. This then gives us x equals 2. We can check this by doing 2 add 6 then divide by 2. This gives us 4.

Common Misconceptions

A. Learners may do the same operations working backwards: 4/2 is 2 and 2 + 6 is 8
C. Learners may use 4 as input when the question gives 4 as the output: 4 add 6 is 10, divide by 2 is 5.
D. Learners may use one inverse operation and not the other: 4 times 2 is 8 and plus 6 is 14.

https://diagnosticquestions.com

Why do this activity?

This is an open-ended activity to accommodate mixed ability classes so learners of all abilities will get practice in finding inverses of different functions. The abler learners can be encouraged to find the algebraic rules for the inverse functions. The slower learners can work entirely with numbers applying the inverses of the component functions one by one in reverse order until they are confident that they understand the process and are ready to progress to finding formulae. Learners are invited to challenge each other to make up their own functions and to be the quickest in finding the input when they are told an output.

Learning objectives

In doing this activity students will have an opportunity to:

- deepen understanding of functions and inverse functions, both simple and compound;
- develop visualization skills linking flow chart diagrams to functions.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images and multiple representations to represent concepts and situations.

Suggestions for teaching

To start, tell the class that Amy and her friends made up the following functions and they think the inputs to their functions that all give the output 10 are 5 for Amy's function, 4 for Busi's, 2½ for Chris's and 5⅓ for Dudu's. Ask your learners "Do you agree?" Ask them to explain why or why not.

Make sure the learners understand how to find inverses either using numbers or finding formulae for inverse functions. Then either copy and distribute the worksheet or write it up on the board. Then circulate asking probing questions while the learners work on the task. Don't give too much help, finding the answers **for themselves** is all important for the learners.

Key Questions

- How can you '**undo**' that function?
- What happens to the output (suggest a number or a letter) if you '**undo**' that operation?
- Would you like to see what happens if your output is the variable x?
- Does it make a difference if you undo this operation before that or change the order? Why or why not?

Follow up

Function Flow https://aiminghigh.aimssec.ac.za/years-9-12-function-flow/
Swop https://aiminghigh.aimssec.ac.za/years-10-12-swop/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.
For resources for teaching A level mathematics see https://nrich.maths.org/12339

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6