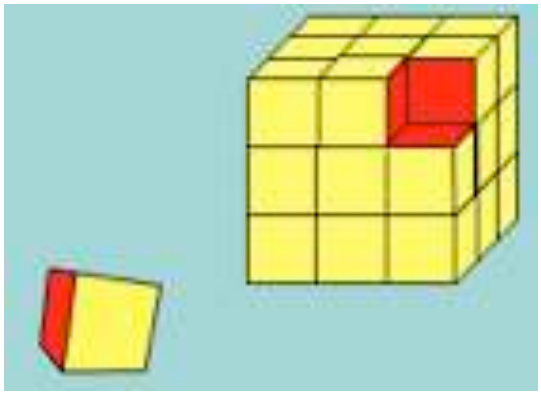
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Title: PAINTED CUBE (Grades 7 to 9)



Imagine a large cube made up from 27 small red cubes each measuring 1cm by 1 cm by 1 cm. What is its volume?

Imagine dipping the large cube into a pot of yellow paint so the whole outer surface is covered. What is its surface area?

Now imagine breaking the cube up into 27 small cubes.
How many of the small cubes will have yellow paint on their faces?

Will they all look the same?

How many red faces and how many yellow faces do they have?

Now imagine doing the same with other big cubes made up from small red cubes.
What can you say about the number of small cubes with yellow paint on?

What interesting discoveries can you make?

SOLUTION

The number of painted faces depends on whether the little cube is at the corner, on the edge, in the middle or buried right inside the big cube. The solutions are given in the table below.

Imagine cubes inside cubes layers of cubes like Russian Dolls or the layers of an onion (see the column 1 in the table below). It is only the outside of the outer layer that is painted yellow. When that layer is removed from the $27cm^3$ there is only one cube in the middle and that will have no red paint on it.

1. Size of large cube	2. No. of small cubes with 6 red faces- inside the original cube	3. No. of small cubes with 5 red faces- on the faces of the original cube but not at the edges	4. No. of small cubes with 4 red faces- on the edges of the original cube	5. No. of small cubes with 3 red faces- at the corners of the original cube	6. Total No. of small cubes. Volume of large cube in cm^3 Total of columns 2, 3 4 and 5	Surface area of large cube in cm^2
3 x 3 x 3	1	6	12	8	27	54
4 x 4 x 4	8	24	24	8	64	96
5 x 5 x 5	27	54	36	8	125	150
6 x 6 x 6	64	96	48	8	216	216
10 x 10 x 10	512	384	96	8	1000	600
23 x 23 x 23	9261	2646	252	8	12167	3174
n by n by n	$(n-2)^3$	$6(n-2)^2$	$12(n-2)$	8	n^3	$6n^2$

To check the formulas we can add them to see if we get the right number of cubes.
$(n-2)^3 + 6(n-2)^2 + 12(n-2) + 8 = n^3 - 6n^2 + 12n - 8 + 6n^2 - 24n + 24 + 12n - 24 + 8 = n^3$.

Notes for teachers

You might like to use this NRICH poster (see link below) or show the large illustration to your learners.
http://nrich.maths.org/content/id/7203/NRICH-poster_PaintedCube.pdf

Why do this activity?

This activity develops learners' visualisation skills and gives them practice in thinking about the volumes and surface areas of cubes. Learners find patterns and teachers can plan for learners of different abilities because some learners will describe the patterns spatially and numerically and the more able (or older) learners can describe the patterns algebraically. Learners will come to see the benefits of keeping a clear record of results, and applying their insights from the first case to ask themselves questions about further cases.

Possible approach

Prepare a 3 by 3 by 3 cube and mark lines on it showing the edges of the 27 small cubes or show the images above to the class.

"Here is a 3 by 3 by 3 cube made up of 27 smaller cubes. What is its volume?"

"Now imagine I dipped it in a pot of yellow paint so that each face of the large cube was covered. What is its surface area?"

"Then after the paint has dried, imagine I split it into the 27 original small cubes. Can you work out how many cubes will have no paint on them? How many will have just one face painted? Or two faces painted, and so on."

*The teacher can **either** draw up a table on the board and tell the learners to copy it and fill it in **or wait** until the learners have made a start to give them the chance to develop their own ways to record their results. Then perhaps they will realise, when it is suggested, that they would have done better to think of making a table for themselves.*

Give learners time to discuss with their partners and work out their answers. While they are working, circulate and observe the different approaches that learners are using, and challenge them to explain any dubious reasoning. Bring the group together to share their responses. Collect the answers for the number of cubes with 0, 1, 2, 3 ? faces painted, and note that they add up to 27. Invite students to explain how they worked it out.

"I'd like you to work on some cubes of different sizes until you are confident that you can always work out how many cubes will have 0, 1, 2 and 3 faces painted. In a while, I'll be choosing a much larger cube at random, and you?ll need to have an efficient method of working it out."

Bring the class together and challenge them to explain how they can work out the number of cubes of each type in a 10 by 10 by 10 painted cube. Depending on the students? experience of working with algebra, you could work together on creating formulas for the number of cubes of each type in an n by n by n cube.

GROUP WORK *You may like to organise your class into groups of 4 to work on this activity.*
Make sure that while groups are working they are reminded of the need to be ready to present their findings at the end, and that all are aware of how long they have left.

Each group should record their diagrams, reasoning and generalisations on a large flipchart sheet in preparation for reporting back. There are many ways that groups can report back. Here are just a few suggestions:

- Every group is given a couple of minutes to report back to the whole class. Other learners can seek clarification and ask questions. After each presentation, learners are invited to offer positive feedback. Finally, students can suggest how the group could have improved their work on the task.

- Everyone's posters are put on display at the front of the room, but only a couple of groups are selected to report back to the whole class. Feedback and suggestions can be given in the same way as above. Additionally learners from the groups which don't present can be invited to share at the end anything they did differently.

- Two people from each group move to join an adjacent group. The two "hosts" explain their findings to the two "visitors". The "visitors" act as critical friends, requiring clear mathematical explanations and justifications. The "visitors" then comment on anything they did differently in their own group.

Key questions

Here are some prompts that could be used if students get stuck:
Where are the cubes with no faces painted?
Where are the cubes with 1 face painted?
Show me a cube with 2 faces painted. Show me a cube with 3 paces painted.
How many of each type of cube would you have in a 4 by 4 by 4 cube?
How many in a 7 by 7 by 7? What about an n by n by n cube?"
How do your algebraic expressions (formulae) relate to the geometry of the cubes?

Possible extension

Partially Painted Cube on the NRICH website provides a suitable follow-up activity.

Possible support

If the learners work in groups the teacher can encouraging learners to take responsibility for ensuring that everyone understands before the group moves on.