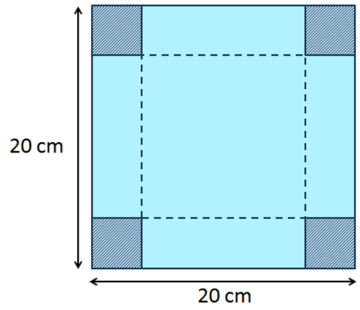
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Title: MAX BOX (Grades 9 to 12)



Take a sheet of paper 20 cm by 20 cm and make a box without a lid by cutting out a square at each corner and folding up the flaps. What is the volume of your box?

Make boxes with different volumes by varying the sizes of the squares that you cut out.

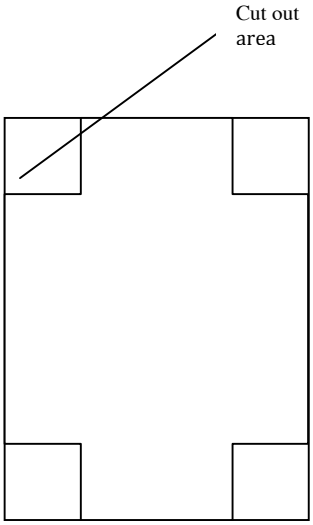
What is the maximum possible volume?
What is the size of the square cut out to produce it?

Now start with different sized square sheets of paper. What is the relationship between the maximum volume and the size of the square cut out to produce it?

Solution

To start with, work out the volume of a cube with a square net of 20cm by 20cm using different sized cut outs.

Here are the results:



1x18x18=324cm³
2x16x16=512cm³
3x14x14=588cm³
4x12x12=576cm³
5x10x10=500cm³
6x8x8=384cm³
7x6x6=252cm³
8x4x4=128cm³
9x2x2=36cm³

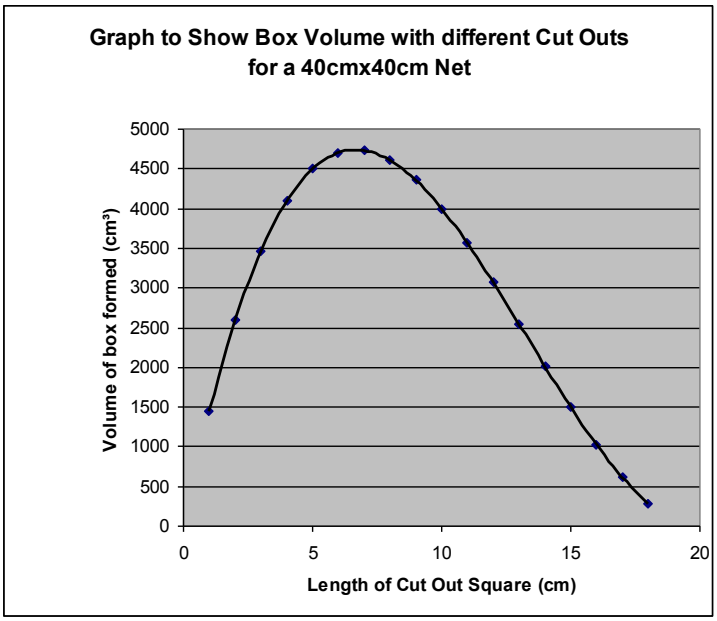
Do the same with nets of sides 40cm by 40cm and 10cm by 10cm.

If x is the length of a side of the square cut out and y is the length of the sides of the net, then:
Volume of the box = $x(y - 2x)^2$
This tells us the volume with different sized cut outs with different sized nets.
For example:
$7(20-14)^2 = 252cm^3$= Volume of the box

x	y=x(40-2x)²
1	1444
2	2592
3	3468
4	4096
5	4500
6	4704
7	4732
8	4608
9	4356
10	4000
11	3564
12	3072
13	2548
14	2016
15	1500
16	1024
17	612
18	288

This table shows the volume of different sized cut outs for a 40cm by 40cm net. We can see the cut out producing the largest volume is about 7. However it is not exactly 7.

Putting the table into the form of a graph makes it easier to understand the values and largest cut out.



So we have found a way to find the cut out producing the largest volume:
Divide the length of a side of the net by 6.

For this particular example:
40/6 = 6.666cm = the length of the cut out square to give the largest volume
Also, for a 20cmx20cm net,
20/6 = 3.333cm to give the largest volume.
The largest volume for a 20cmx20cm net is therefore $592.5cm^3$

Notes for teachers

Why do this activity?

This practical activity helps learners to visualise and consolidate the formula for working out the volume of a cuboid. It also gives them an the experience of investigating relationships between variables, making conjectures, testing out ideas and comparing different methods for getting the best solution.

Possible approach

You could start with a mathematical thinking and visualisation experience. Say to the learners: "Close your eyes. Now imagine a 20 by 20 square sheet of paper. Think about that square. Now imagine cutting out a square of side 4cm from each corner, and folding up the flaps. Turn to your neighbour and check you agree what the dimensions that the resulting box will be. Try to do this without writing anything down or using your hands to gesture."

Bring the class together and confirm the dimensions, and the resulting volume.

You can choose to give each pair of learners sheets of scrap paper and scissors and ask them to make boxes and this would be the best plan for younger learners.

Alternatively, you could make this an exercise based on entirely on visualisation. Allow some time for the learners to work in pairs to explore the effect of different sized cuts on the volume of the resulting box. Can they find the largest possible volume?

Collect the results and list them in an ordered table. Ask about any noticeable patterns or trends. What is the maximum volume that anyone has found? Can we improve on this?

There are different possibilities for this:
some learners may use trial and improvement, possibly using calculators;
some learners may draw up a spreadsheet and use trial and improvement;
older and more able learners may draw up a graph and look for maximum values.

According to your class encourage some learners to try each method so that you can bring the class together to compare and discuss their results.

Ask pairs of learners to choose a different sized starting square (for example squares of sides: 10 cm, 12 cm, 18cm, 20cm, 24 cm, 30 cm, 36 cm and 40 cm) and to find the cut that produces the maximum volume.
Collect the results on the board and ask for any comments, and what patterns they notice.

Key questions

Explain how you find the volume? Can you write that down as a formula?
How can you be sure you have found the maximum volume?
Can you convince someone else?

Possible extension

Instead of starting with square sheets of paper, learners may investigate rectangular ones. In order to make pattern spotting easier, you may wish to organise the activity in some way, for example giving different groups of students sets of rectangles (such as rectangles where the length is twice the breadth, three times the breadth, four times the breadth etc.)

Possible support

It will help learners who are struggling to use squared paper measuring 20 squares by 20 squares and to cut out and make boxes of different volumes recording their results for each one.