

M, M and M

3 Can you find all the sets of five positive whole numbers that have

3 a mean of 4

3 a median of 3

3 and a mode of 3?

5 One solution is 3, 3, 3, 5 and 6.

6 Can you be sure that you have found all possible solutions?

If I also tell you that the range is 10 can you now find one solution?

See <http://nrich.maths.org/6267>

SOLUTION

We have to find 5 positive whole numbers that have a mean of 4, a median of 3, and a mode of 3.

Because the median is 3, we know that if we write the numbers in numerical order, the middle number has to be 3.

Because the mode is 3, there must be at least one other 3.

1st number: must be equal to, or less than 3

2nd number: must be equal to, or less than 3

3rd number: must be 3

4th number: must be equal to, or greater than 3

5th number: must be greater than 3 (if the largest number was 3, the mean could not be 4)

Since the mean is 4, we know that all the 5 numbers must add up to 20 (20 divided by the number of numbers is 4). We can guarantee two 3s in the set, so the remaining numbers must total 14.

Now we need to make a list of all the possible sets whose starting number is less than 3, and (apart from 3) no numbers are repeated twice. The only time that this repetition would work, would be with three 3s, and two of another number, but no whole number multiplied by 2 equals 11 (20 minus three 3s).

Here is the list:

1, 2, 11

1, 3, 10

1, 4, 9

1, 5, 8

1, 6, 7

2, 3, 9

2, 4, 8

2, 5, 7

3, 3, 8

3, 4, 7

3, 5, 6

Written with the other numbers, the full sets are:

1, 2, 3, 3, 11

1, 3, 3, 3, 10

1, 3, 3, 4, 9

1, 3, 3, 5, 8

1, 3, 3, 6, 7

2, 3, 3, 3, 9

2, 3, 3, 4, 8

2, 3, 3, 5, 7

3, 3, 3, 3, 8

3, 3, 3, 4, 7

3, 3, 3, 5, 6

Note: 1, 7, 6 = 1, 6, 7
because ultimately the
numbers will be written
in ascending order.

There are eleven sets. We have found all possible sets because we wrote them in ascending order, with the smallest possible number in first position, then the next smallest in second position and the remainder in the last position. After that we continue to write the smallest number in first position till we had exhausted all possibilities for the other two numbers, then moved on to the second smallest number in first position, and so on. Finally we deleted all repeating combinations, and all the final sets satisfy all the conditions.

If the range is 10 the numbers must be 1, 2, 3, 3, 11, because this is the only set with a range of 10.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

You are given data on the lifespan of two different batches of lightbulb, manufactured at the same company. By looking at the data, decide which of the statements below is true.

Batch A

Mean: 1100 hours

Range: 50 hours

Batch B

Mean: 800 hours

Range: 400 hours

A “On average, batch B’s lightbulbs last a shorter length of time and are more consistent”

B “On average, batch A’s lightbulbs last a shorter length of time, but are more consistent”

C “On average, batch A’s lightbulbs last a longer length of time, and are less consistent”

D “The lifespans for batch A’s lightbulbs are more similar to each other than batch B’s”

D. is the correct answer.

Common Misconceptions

A. No, more consistent means a smaller range

B. No, batch A’s light bulbs last a longer time.

C. No, although batch A’s light bulbs do last a longer time they are more consistent.

<https://diagnosticquestions.com>

Why do this activity?

This problem offers the learners an opportunity to consolidate what they are expected to know about mean, mode and median whilst also challenging them to work systematically, and justify their reasoning.

Intended learning outcomes

Revision of concepts of mean, median and mode.

Suggestions for teaching

Start with the diagnostic question.

Then write five numbers on the board: 5, 3, 6, 3, 3 and ask for the mean, median and mode of this set. Resolve any disagreements. “OK this is too easy for you, so what if I told you that the mean, mode and median of 5 positive whole numbers were: mean=4. mode=3. median=3. Would you have been able to tell me the five numbers?”

Allow some time for **individual work** and ask learners to come and write the set of numbers they have found on the chalkboard as soon as they find them if they are not already listed. Then ask “Are there any other sets of five numbers that fit these conditions?”

Collect a few suggestions and then ask “There seem to be quite a few, can you find some more?” “Can you find them all?” Allow some time for **individual work** then ask “Can you convince yourself that you have found them all?”

Then suggest that the learners work **in pairs** and ask them “Can you convince your partner that you have found all the solutions?”

Then suggest that the pairs join up to make **groups of 4 learners** and the groups list all their sets of solutions in a systematic order that shows that the list must be complete. Ask “Can each group convince the rest of the

class that they have all the solutions?” Tell them that you will pick one member from each group at random so every member of the group has to be ready to report the results for the group and to explain why they think that they have found ALL possible solutions.

Invite early finishers to list their sets of solutions on the board in a way that makes their system explicit. You may end up with several different orderings of the solution sets.

You could either: invite someone from each group to explain their logic to the class, OR invite the class to work out what the reasoning is behind each ordering and ask the group to confirm, OR ask learners to list just the first few sets of numbers and ask the class to predict which other sets will follow.

Next, say “If I had also told you that the range of these 5 numbers was 10, how many solutions would there have been?”

Finish the activity by asking learners to make up a similar question including mean, mode, median and range for their partner. Can they find a question of this sort that has a unique solution?

Key questions

Which piece of information is the most useful to start with?

If you know that the mode is 3 what does that tell you about the number of 3s in the set?

If you know that the mean of 5 numbers is 4 what is the total of the 5 numbers?

What process allows you to be confident that you will have found all the results by the end?

Possible extension

Could we have chosen a mean, mode and median for which there was only one solution?

Could we have chosen a mean, mode and median for which there were no solutions?

Would your answers be different if you were allowed to include negative numbers?

Could you make a list of 6 numbers and still have the same mean, mode, median and range?

Could you make a list of 7 numbers and still have the same mean, mode, median and range?

What about a list of 8 numbers (or more)?

Possible support

Learners who find it difficult to work systematically could write each solution set on a separate piece of paper and try to arrange the pieces of paper into ‘families’ that is groups sharing a characteristic, then arrange the families in order.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.

Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not included in the school curriculum for Grade 12 SA.**

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6