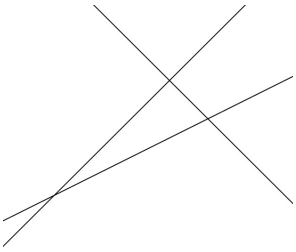
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH



GRAPHICAL TRIANGLE

What is the area, (in square units) of the triangle formed by the three lines whose equations are:
$y - x = 6$,
$x - 2y = 3$ and
$x + y = 6$?

METHOD 1
Plot the graphs.
Find the coordinates of the vertices of the triangle.
Box in the triangle.

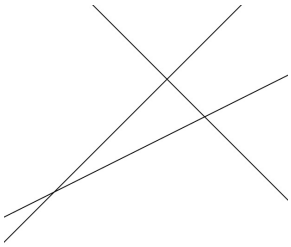
Calculate the areas of all the triangles in the box.

METHOD 2
Plot the graphs.
Find the coordinates of the vertices of the triangle.
Explain how you know the triangle is right angled.
Calculate the lengths of 2 edges of the triangle.

Help

Use axes with x from -20 to +10 and y from -15 to +15 and to plot the lines:
$y - x = 6$, $x - 2y = 3$ and $x + y = 6$?

The three lines will look like this.



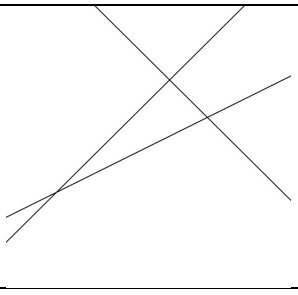
Imagine a box in the diagram going through the 3 vertices.
Use the box to find the area of the triangle made by the 3 lines.

Write out a 'to do' list that you could follow step by step to find the area of the triangle.

Extension

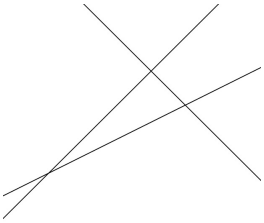
Find the area by a different method.

Odd one out https://aiminghigh.aimssec.ac.za/years-7-to-9-odd-one-out/



NOTES FOR TEACHERS

SOLUTION

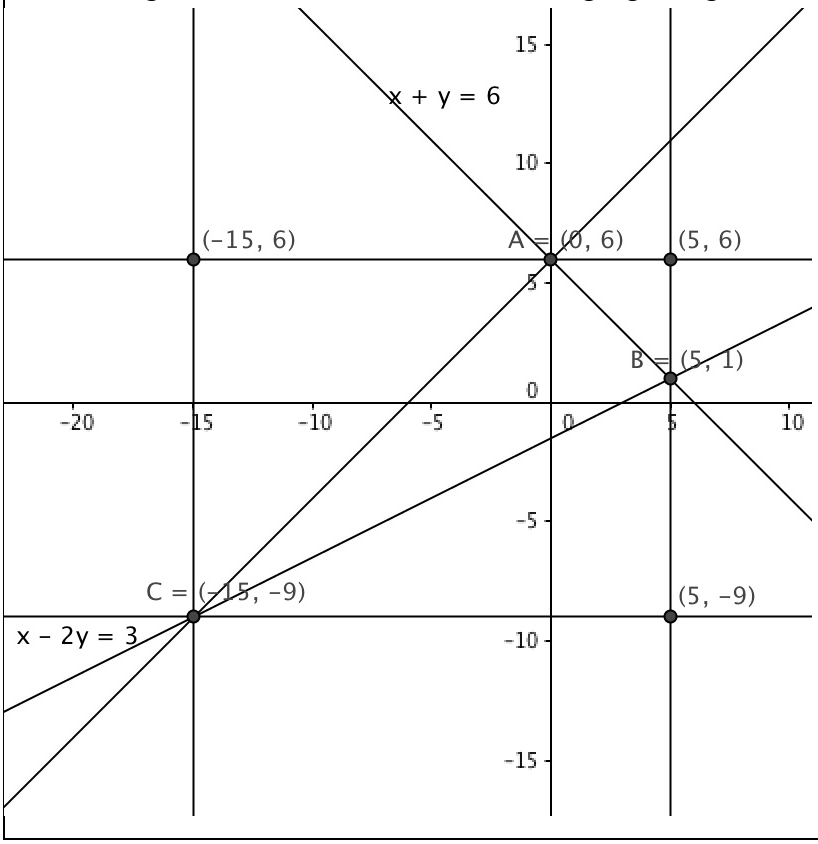


Draw the graphs of the lines: $y - x = 6$, $x - 2y = 3$ and $x + y = 6$?
Take the three lines in pairs and solve their equations simultaneously to find the coordinates of their points of intersection, i.e. the coordinates of the vertices of the triangle. If the coordinates are found graphically they can be checked by substitution in the equations avoiding the need for solving the equations algebraically.

METHOD 1

The coordinates of the intersections are A(0,6), B(5,1) and C(−15,−9).

The area of triangle ABC can be found by enclosing the triangle in a rectangle 20 units by 15 units, and subtracting the areas of the three surrounding right angled triangles from the area of the rectangle.



Area of rectangular 'box' = 20 × 15 = 300

Areas of outer triangles:
½(15 ×15) = 112½
½(5 × 5) = 12½
½(20 × 10) = 100

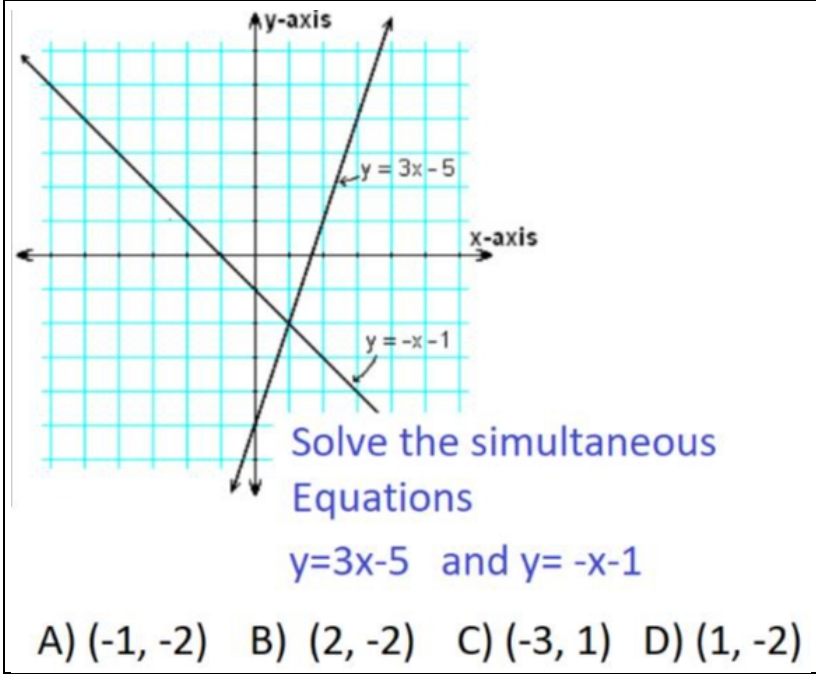
Area of inner triangle ABC:
300−(112½+12½+100)=75.

METHOD 2

Note that y= -x + 6 and y = x + 6 are perpendicular, so angle BAC is 90^o.
By Pythagoras Theorem or using the distance formula:
$AB^2 = 50$ and $AC^2 = 450$.
So Area ABC = ½√50 × √450
= ½ ×150
= 75

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is D.
It's easier just to read off the coordinates from the graph where the lines meet. Emphasise that learners should always check if the x value works and produces the y value in BOTH equations.
Common Misconceptions
A. Confusion over signs**.**
B. Probably a guess
C. Probably a guess

https://diagnosticquestions.com

Why do this activity?

This activity can be done with or without technology. The free graphing software package GeoGebra for computers or phones can be downloaded from www.**geogebra**.org together with instructions on how to get started and many maths apps and lesson suggestions.

This activity can be an exercise in plotting graphs of straight lines, or in solving pairs of simultaneous linear equations. It involves problem solving (how do we find the area) and also complex procedures (combining several routine procedures). Teachers can choose how much direction to give and how much to leave to the learners, according to their age and ability and to how much experience they already have of these tasks.

The area can be found by 'boxing' in the triangle and subtraction, or by recognition that two of the lines are perpendicular and using Pythagoras Theorem to find the lengths of the edges.

Learning objectives

All years – practice in problem solving
Years 9 and 10 This could be revision of plotting straight lines and of area and an introduction to simultaneous equations before introducing the algebraic method.
Years 11 and 12 Revision of straight lines, area, conditions for perpendicularity, distance formula in analytic geometry, Pythagoras Theorem and simultaneous equations.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.
In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **apply** knowledge and skills;
- **visualization** - develop the skill of interpreting and creating visual images to represent concepts and situations.

Suggestions for teaching

Decide whether you want the learners to work in pairs or small groups to discuss the problem and to use their own methods,
or give learners the choice of plotting the 3 lines or of solving the 3 pairs of equations before asking them to find the area, or tell them which method to use if you want them to have practice in one method or the other.

Either write the problem on the board or give out the worksheet and squared paper.

(1) Tell the learners to work in pairs or small groups to solve the problem. Circulate asking probing questions (see Key Questions below) and/or, after about 15 minutes, have a class discussion about the possible methods and write on the board a 'to do' list for each method.

(2) Ask the learners how they would find the coordinates of the points where the lines intersect (vertices of the triangle). If both methods are suggested (plotting the lines and solving pairs of equations) then either tell them to choose a method for themselves, or tell them which method to start with.

(3) When most learners have found the coordinates of the vertices of the triangle make sure they understand that having found 2 points on each line they can join the points to plot the lines. Draw the diagram on the board and check that all the learners have recorded it correctly. Then lead a class discussion about how to find the area of the triangle.

(4) You could decide that some learners should use Method 1 and other learners should use Method 2 and then learners should explain their work to the whole class.
Learners might think of 'boxing' in the triangle as this method is frequently used for proving the area formula and also for finding areas of triangles.
If they want to use ½base x height then they need to look for right angles before choosing the base.
If both methods are suggested then let the learners choose which method to use, if they only suggest one method then that can be the one they all use.

After trying to ensure that they will know how to go about it, you might ask learners to find the area for homework.

Finally check the work with the whole class. Go over common errors and summarise what has been learned.

Key questions

- Where does the line cut the y axis?
- Where does the line cut the x axis?
- With these 2 points have you got enough information to plot the line?
- What is the gradient of that line? (demonstrate with pencils or sticks)
- If you know the equation of the line, how do you find the gradient?
- Does the line you have drawn have the right gradient?
- Where do the lines intersect?
- What do you need to know to find the area of that triangle?
- Can you see any right angles? How do you know that is a right angle?
- How could you find the length of that line?
- What do you need to know to find the area of a triangle?
- If you use that formula, where is the base and where is the height?

Follow-up ideas

Odd one out https://aiminghigh.aimssec.ac.za/years-7-to-9-odd-one-out/
Solve the triangle https://aiminghigh.aimssec.ac.za/years-11-to-12-solve-the-triangle/
Kissing triangles https://aiminghigh.aimssec.ac.za/years-9-11-kissing-triangles/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6