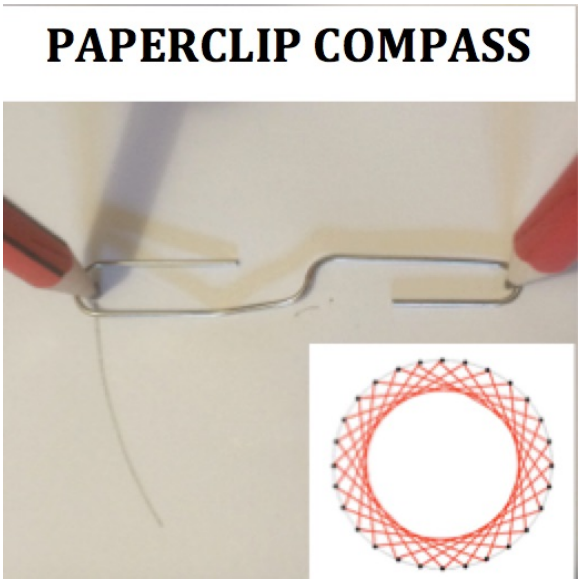
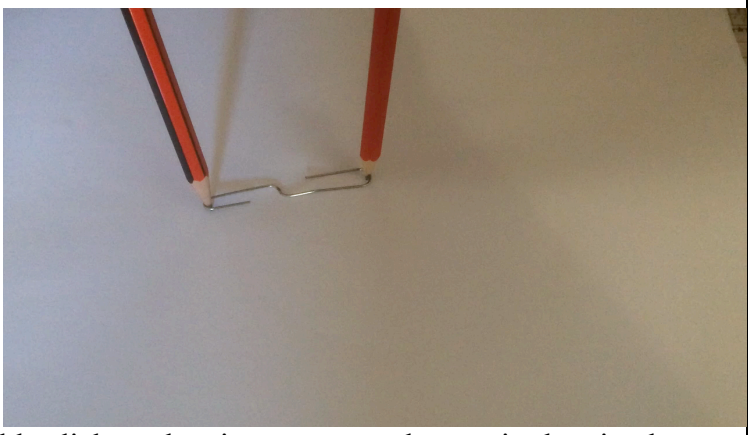
CLOCK ARITHMETIC AND ENVELOPES

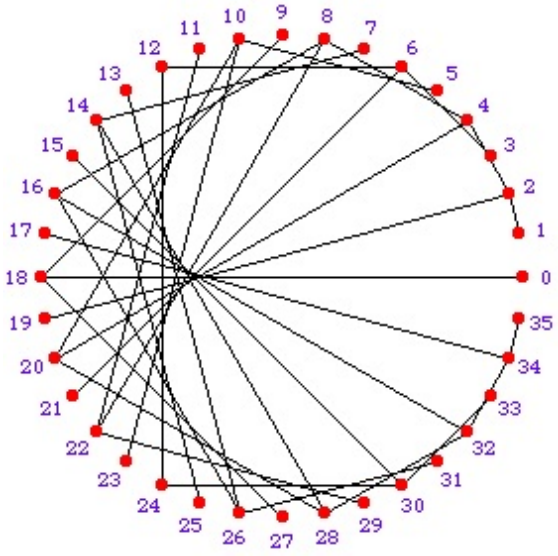




Double click on the picture to start the movie showing how to draw a circle using the paperclip compass.

Or go to: https://youtu.be/iewxclQEAnk

Cardioid technology is used in microphones and speakers. Cardioids appear in the motion of planets and in many other ways. See this YouTube video. www.youtube.com/watch?v=tTYGLoVb5xA



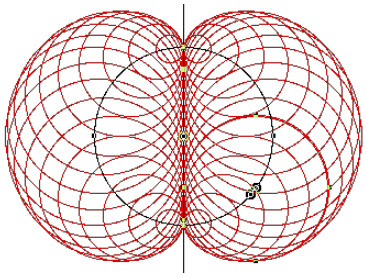
To draw a cardioid, first draw a circle and carefully mark 36 points around the circle at 10 degree intervals and number the points n = 0 to 35. You are going to draw cardioids and other envelopes by joining each point n to the point given by some function of n.

1. Join n →2n using clock arithmetic.

You already use clock arithmetic in:
12s with months and with hours;
in 60s with minutes and seconds;
also in 7's with days of the week.

Double each number and join it to the point given by the remainder after dividing by 36 (clock arithmetic). For example 19 is joined to 2 because 2 times 19 is 38 which is 1× 36 + 2. Similarly 25 is joined to 14 because 2 × 25 is 50 which is 1× 36 + 14.

2. This shape is called a nephroid. Start with a base circle and a diameter of that circle. Construct the nephroid by drawing circles with centres on the base circle and each tangent to the diameter of the base circle.



The cardioid and nephroid are examples of envelopes which are curves that are tangent to a family of curves at each point of contact.

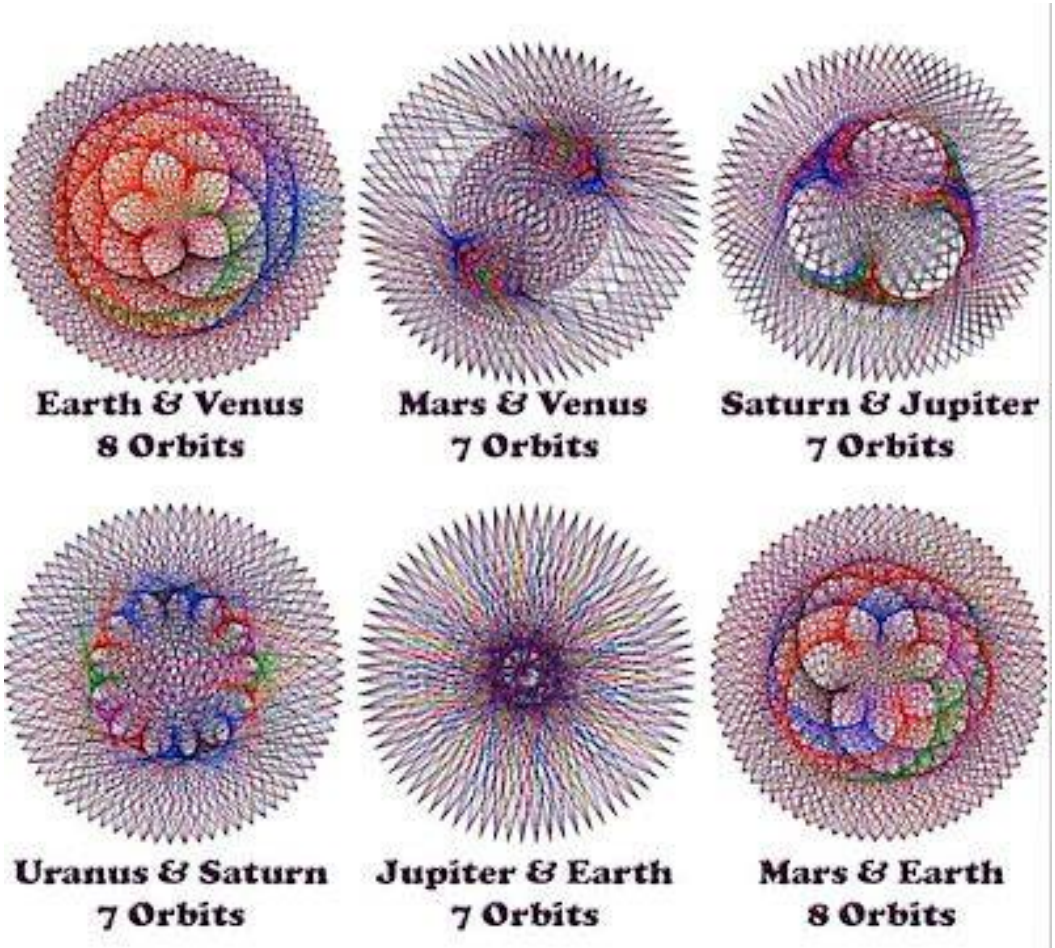
3. Experiment with different patterns using Method 1 and other functions, for example n →3n and n → 2n + 5 etc.

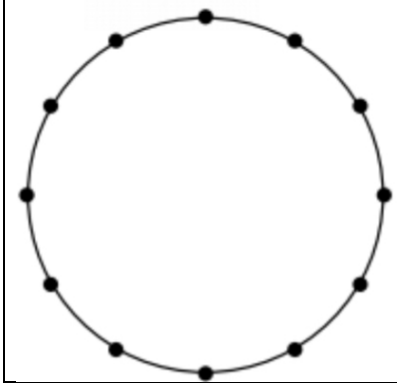
THE DANCE OF THE PLANETS

If you join the positions of pairs of planets in our Solar System at different times you get these patterns.

This happens because the motion of the planets around the sun is periodic.

The video (link above) shows the pattern for the Earth and Venus.





Help

If you are struggling to measure angles and mark points on a circle you could start with this circle with 12 points already marked and join each point n to $2n$ where, like a clock, the point 10 is joined to 8 because $2\times10 = 20 = 12 + 8$

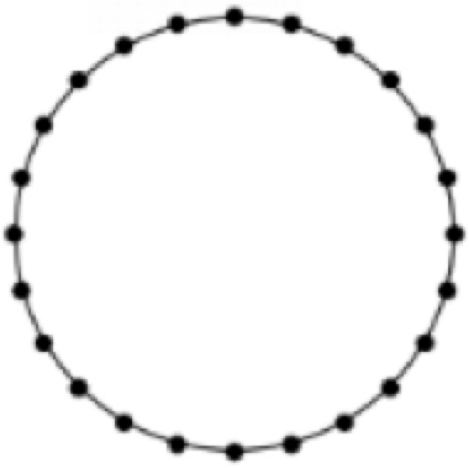
Then try 6 points or 24 points (see pages 5 and 6).

When you succeed in drawing patterns with a few points you might find it easier to copy one of the designs rather than working from a formula.

Extension

Mystic Rose
https://aiminghigh.aimssec.ac.za/years-7-12-mystic-rose/

A different construction – Join every point to every other point around the circle. How many lines do you need to draw?



NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

If today is Tuesday what day of the week will it be in 30 days time?

A. Tuesday **B.** Sunday **C.** Thursday **D.** Friday

The correct answer is C.

Some learners will realise that it will be Tuesday again in 7, 14, 21 and 28 days and, in general, when the number of days is a multiple of 7. In this quiz they just need to add 2 more days so the answer is Thursday.

Some learners will count off 1 to 30 going through the days of the week. Learners who give any other answer may have miscounted or just guessed.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners practice in using a ruler, protractor and compasses to draw accurate geometrical constructions. If compasses are not available then a paperclip can be used. To make the patterns learners will need to follow instructions, to measure lengths and angles, to draw circles, and to work accurately. The activity offers opportunities for talking about the geometrical properties of the shapes.

The activity could be developed into a class research project to investigate the connection between polygonal, circular, cardiod, nephroid and other envelopes and the functions that lead to the different patterns.

Teachers can plan for learners of different abilities by giving learners different patterns to draw, for example slower learners could be given a circle with points already marked on the circumference to start with. The activity may improve learners' attitude to mathematics by appealing to some learners who do not like mathematics and to others who find it difficult. The activity encourages creativity. It offers opportunities for talking about the geometrical properties of the shapes and also for thinking mathematically about the classification of the patterns.

Intended learning outcomes

Measuring Angles: Accurately use a protractor to measure and classify angles: $< 90^{o}$ (acute angles); Right-angles; angles $> 90^{o}$ (obtuse angles); Straight angles; $> 180^{o}$ (reflex angles).
Constructions: Use a compass, ruler and protractor appropriately to construct geometric figures accurately, including: angles, to one degree of accuracy and circles.

Generic Competences

This activity requires learners to:

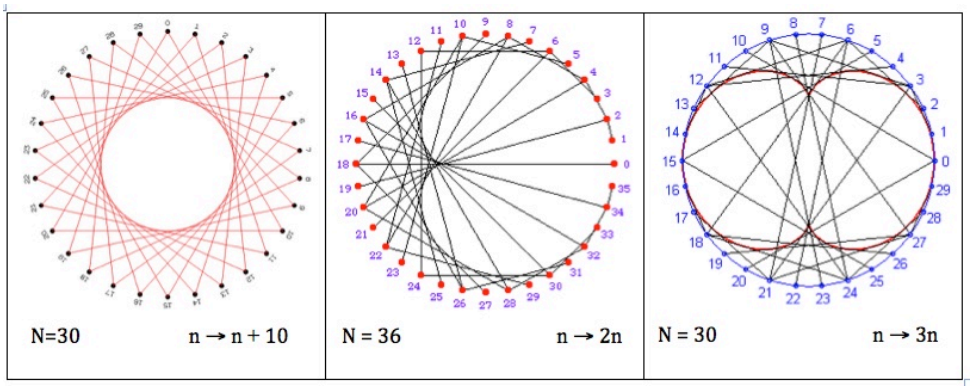
1. develop the skill of interpreting and creating visual images to represent concepts and situations;
2. be creative and innovative - to apply knowledge and skills;
3. be able to communicate in writing and speaking
   a. co-operate - to collaborate/work in a team
   b. to communicate, exchange ideas, criticise, and present information and ideas
   c. to analyze, reason and communicate effectively.

Suggestions for teaching

Resources needed. Pencils, rulers, protractors and compasses (or paperclip compasses).
Start with the Diagnostic Quiz. Ask several students if they know a quick and easy may to work out the answer. Practice with other examples about days of the week, months, hourse, minutes and seconds and explain that this method is known in schools as clock arithmetic. (You could tell the learners that the correct mathematical name is modulus arithmetic).

Start with the students working individually as it is important for sudents to read questions for themselves. When some students have drawn patterns of their own and about half the class have finished parts 1 and 2 you could ask the learners to show their designs to the class and explain how they produced them.

Circle **Cardioid** **Nephroid**



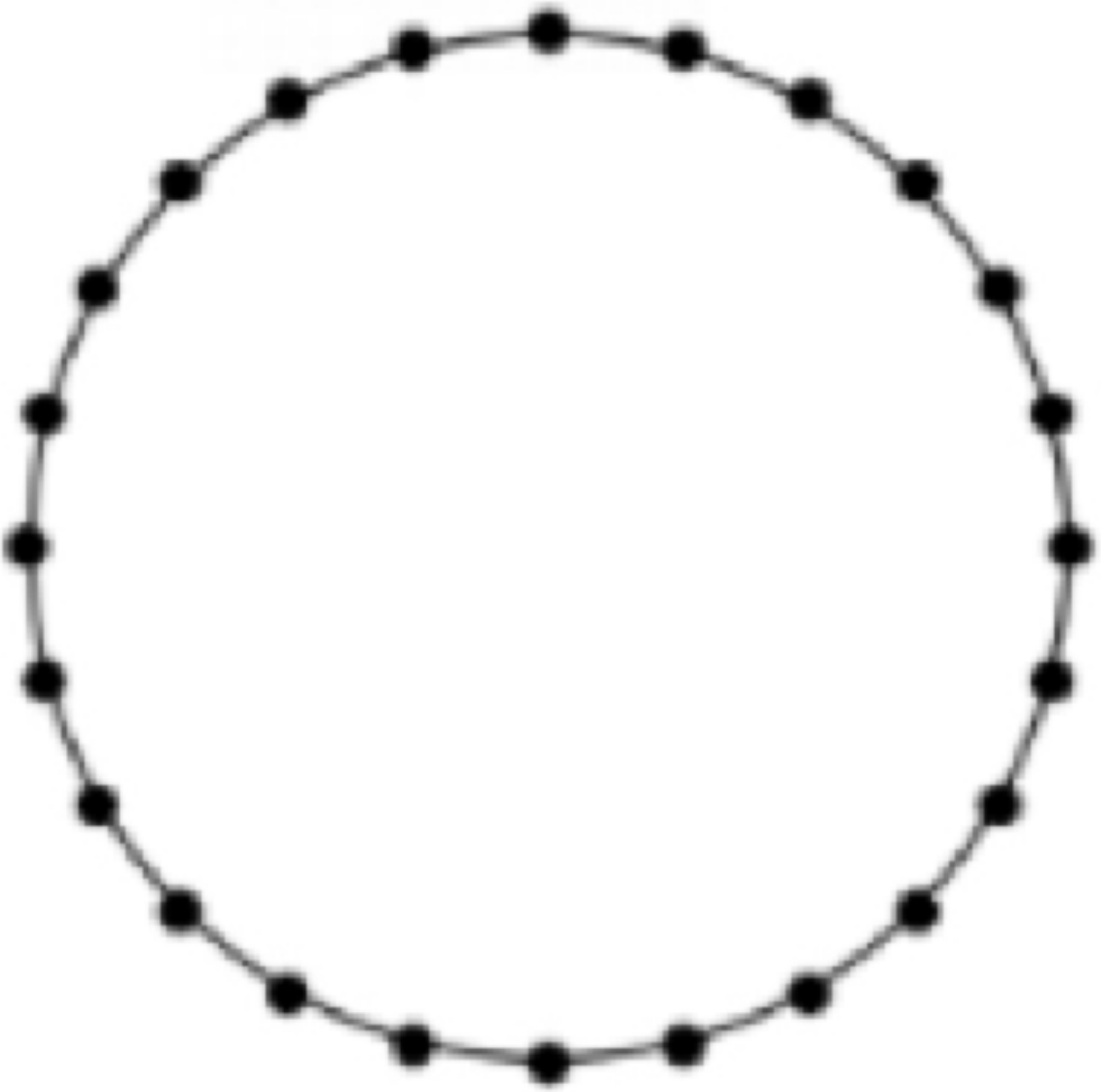
You could then show this picture to the class (see page 7) and organise a class project. Ask the students to compare the patterns and explore possible rules that might relate to the types of envelope. Let the students work in groups and create their own posters to put on display on the classroom wall.

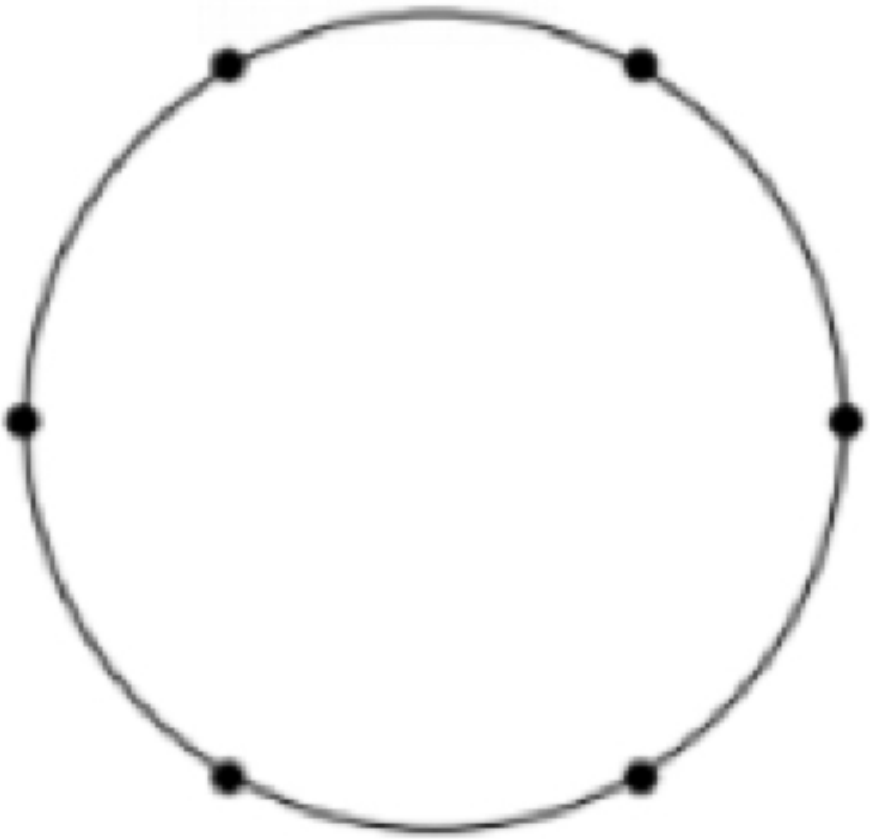
Key questions

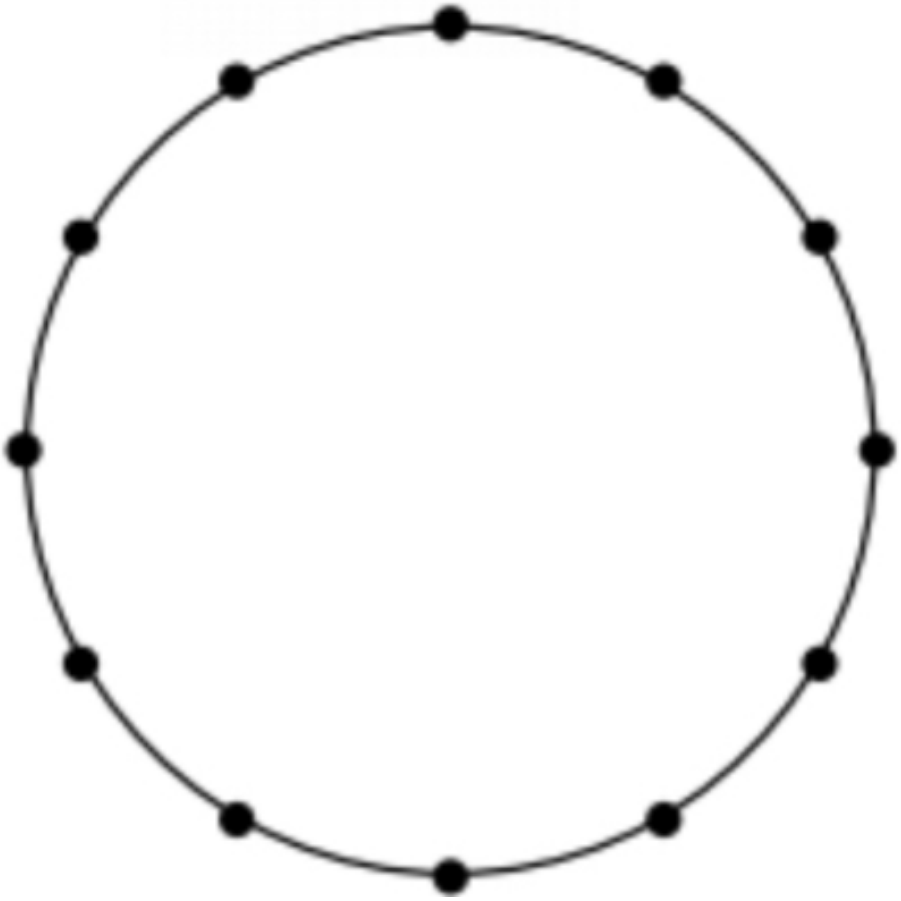
- What pattern would you get if you joined every 8th point on the 24 point circle?
- What pattern would you get if you joined every 6th point on the 24 point circle?
- What pattern would you get if you joined every 4th point on the 24 point circle?
- What pattern would you get if you joined every 3th point on the 24 point circle?
- What angle would you use to mark 30 points around the circle?
- What points would you join on the 30 point circle to get a hexagon?
- What points would you join on the 30 point circle to get a pentagon?
- Could you make a square by joining equally spaced points on a 30 point circle? Why or why not?
- What pattern would you get if you joined every 9th point on the 30 point circle?
- Does your pattern have any symmetry?
- **And looking at a display of many patterns drawn by the members of the class:**
- Do you notice anything about the points that are joined that give polygons?
- Do you notice anything about the points that are joined that give polygons?
- Do you notice anything about the points that are joined that circle envelopes?
- Do you notice anything about the points that are joined that give cardiod envelopes?
- Do you notice anything about the points that are joined that give nephroid envelopes?
- Do you notice anything about the points that are joined that give other envelopes?

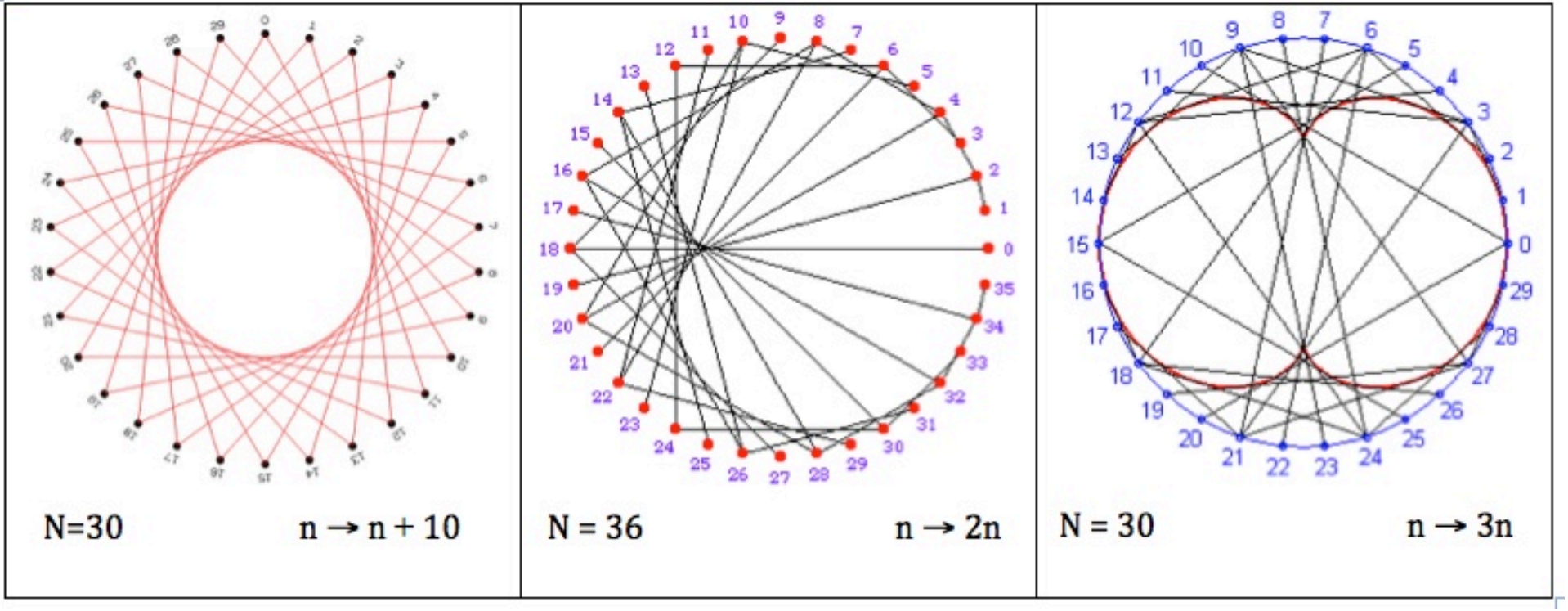
Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6








N=30
n → n + 10
N = 36
n → 2n
N = 30
n → 3n