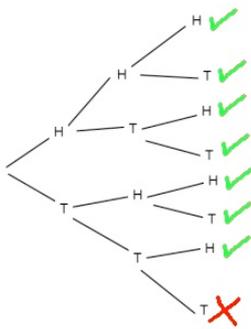## Grades 8 to 10 AT LEAST ONE

If Busi has 3 children what is the probability that at least one will be a girl?

Imagine tossing a coin three times.

What's the probability you will get a head on at least one of the tosses?

Suppose you drew this tree diagram. How would it help you to find the answers to these questions?

What is the connection between the questions?

How could you work out the probability of Busi having 3 boys?

What do you notice about the probabilities of having at least one girl and having 3 boys?

Solution

The probability of a girl is ½ and the probability of a boy is ½.

This is the same as the probability of a head or a tail when you toss a coin.

So the coin tossing is a way of mathematically modeling the real life situation of probabilities for different numbers of boys and girls in a family.

The tree diagram shows the results for 3 tosses of a coin with a tick against all the branches showing at least one head and a cross on the branch showing no heads (all tails).

We can write the probability ½ on each of the 14 small branches of the tree which gives the probability for each outcome as ½×½×½=⅛.

We can list the possibilities as HHH, HHT, HTH, HTT, THH, THT, TTH and TTT and all these outcomes have probability ⅛.

So there is at least one head for 7 out of 8 of the outcomes and the probability of getting at least one head is ⅞.

The probability of getting 3 tails is ½×½×½=⅛ which is 1 – probability of at least one head.

These two probabilities add up to 1.

We can list the families in a similar way: GGG, GGB, GBG, GBB, BGG, BGB, BBG and BBB and the same reasoning shows that the probability of at least one girl is ⅞.

Notes for Teaching

Why do this problem?

This problem can be used to introduce tree diagrams or to develop a better understanding of tree diagrams. Learners can easily relate to the situations of children in a family and tossing a coin and this question offers an opportunity for mathematical talk. Even if they have never seen a tree diagram before, the class can be given the diagram with the ticks and one cross and asked how they might use it to help them to answer the questions. The activity offers opportunities to use different methods, for example listing all the possibilities. Rather than starting by the class tossing a coin to get an experimental probability they are encouraged to think mathematically about using this idea to model the real life situation.

Possible approach

Teachers could draw the tree diagram and write the questions on the board and ask the class to try to answer the questions. The class could work in pairs or small groups. After allowing sufficient time the teacher can lead a discussion asking learners to make suggestions about using the tree diagram. Then, based on what the learners have done and what they say about it, the teacher can give her explanation summarize what they have learned.

Key questions

What is the probability of a baby being a girl?

What is the probability of the coin landing as a head?

What happens for two tosses of the coin one after the other? Can you list the outcomes?

How does the tree diagram show the results for tossing a coin 3 times?

Possible extension

Learners could extend the question to 4 tosses or more.

Possible support

Listing all the outcomes provides an alternative method and then the learner can trace each outcome along 3 successive sub-branches of the tree.