REPETITION

Multiply 0.12121212… by 100 and you get

12.12121212…

Subtract the smaller number from the bigger one and you get

$99 \times 0.121212\ldots = 12$

What does this tell you about the fraction $\frac{4}{33}$ in the picture?

The decimal point is used here. Those who prefer the decimal comma can write the decimals accordingly.

How could you check your result in a different way?

Find the decimal equivalent of the fraction $\frac{4}{33}$ by dividing 4 by 33.

Now look at the two recurring decimals

0.44444...

4.44444...

Multiply the first number by 10. What do you get?

Subtract the smaller number from the bigger one. What do you get?

Can you use these two answers to find the common fraction that has the decimal equivalent 0.44444...?

To go from the recurring decimal to the fraction you multiply by 10, 100, or 1000 according to the number of digits that repeat themselves so that, when you do the subtraction, you get 9 or 99 or 999 … times the decimal equal to a whole number.

Now match the following recurring decimals with their fraction equivalents and use the methods described above to check your answers.

0.151515...	$\frac{7}{9}$	0.454545...	$\frac{2}{3}$	$\frac{1}{7}$	0.777777...
$\frac{26}{111}$	0.666666...	0.14285714...	$\frac{5}{33}$	0.234234...	$\frac{5}{11}$

HELP

Do the 'Divide Divide' activity first https://aiminghigh.aimssec.ac.za/years-7-9-divide-divide/

You divide the numerator by the denominator to turn fractions into decimals. To find the decimal equivalent of $\frac{7}{9}$ divide 7 by 9. To turn decimals into fractions multiply the fractions with a single repeating digit by 10, with 2 repeating digits multiply by 100, with 3 repeating digits by 1000 … etc

NEXT

Make up your own set of 6 matching pairs and put them, jumbled up, into a table like the one in this question. Exchange your set with another learner. Each of you should check and match up the set created by the other learner.

GUIDE FOR HOME LEARNING

SOLUTION

$100 \times 0.12121212\ldots = 12.12121212\ldots$

$12.12121212\ldots - 0.12121212\ldots = 12$

So $99 \times 0.12121212\ldots = 12$ and

$0.12121212\ldots = \frac{12}{99} = \frac{4}{33}$

To check this result we can divide 4 by 33 and this gives the recurring decimal 0.12121212…

Now with the two recurring decimals

$10 \times 0.44444\ldots = 4.444444\ldots$

$4.44444\ldots - 0.44444\ldots = 4$

So $9 \times 0.44444\ldots = 4$ and $0.44444\ldots = \frac{4}{9}$

In the table below the two representations of the numbers are matched in pairs.

0.151515...	0.666666...	0.454545...	0.14285714...	0.234234...	0.777777...
$\frac{5}{33}$	$\frac{2}{3}$	$\frac{5}{11}$	$\frac{1}{7}$	$\frac{26}{111}$	$\frac{7}{9}$

Why do this activity?

This activity should follow on after the 'Divide Divide' learning activity https://aiminghigh.aimssec.ac.za/years-7-9-divide-divide/ which is all about converting common fractions into their decimal equivalents. The 'Repetition' activity is about the reverse process of converting decimals to common fractions.

The Repetition learning activity can be used for learners to discover **for themselves** how and why the process of converting recurring decimals to common fractions will always work.

Learners may need to repeat this method many times with the aim that they **make sense of** the process. Each time ask searching questions (see Key Questions below) to help learners to think mathematically so they come to **understand** the process and to be able to explain how it works. Do **not** do this for the learners but instead guide them to do it for themselves step by step.

Learning objectives

Having done this activity learners should know how to convert a recurring decimal to a fraction and understand why the process works.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- exchange ideas, criticise, and present information and ideas to others.

GUIDE FOR HOME LEARNING

As with all these Guides for Home Learning, this guide relates to learners of different ages. Deciding exactly how to engage with the activity is up to the discretion of the adult who is guiding the young people (or it might be an older sibling helping younger people). This activity is suitable for learners of ages 11 to 17.

Often very intelligent adults say "I never did understand fractions at school". That is a sad commentary on how they were taught. Beware of this pitfall with the Repetition activity. Your aim should be for the learners to make sense of the process and to feel that they understand why it works. Very often learners hate fractions. This is usually because they have been taught (for example in adding fractions) to 'turn handles' and produce answers using methods they do not understand.

'Do it like this' are **not** words that teachers should say to learners, but it is good if learners can explain to others WHY they 'do it like this'.

So start the session by writing down the two numbers 0.121212… and 12.121212… and ask 'which is bigger?', 'which is smaller?', 'what do you notice about these numbers?'

Then ask the learners to multiply the smaller number by 100 and write down the answer. What do they get? What do they notice? **Do not allow calculators.**

Then ask them to subtract the smaller of the two numbers from the larger one. What do they get?

Ask if they could use what they have done to make the smaller number into a fraction written as one number divided by another.

If you have a group, then give the learners time to talk to each about it. Share ideas. If they are not confident about this process then ask:
1. 'Can you find 99 times the smaller number?' They should arrive at the conclusion that 99 times 0.121212… is equal to 12.
2. 'Can you write this expression in a different way?'
3. 'Can you use this to write the smaller number as a fraction with one number divided by another?',
4. 'What does this tell you about the fraction $\frac{4}{33}$?'
5. 'How could you check this result in a different way?'

Do the division together… 33 divides into 40 once with remainder 7, 33 divides into 70 twice remainder 4 etc …. Do not allow the use of a calculator except to check an answer AFTER doing the calculation.

Either do the second example 0.444… step by step together getting the learners to say what to do at each step, or give them time to do it individually. Then discuss it as a group.

Then set the matching exercise for the learners to do, perhaps in a later session another day. Finally check their answers, summarise what they have learned and do the Diagnostic Quiz.

Key Questions

- Which number is bigger?
- Which is smaller?
- What do you notice about these numbers?

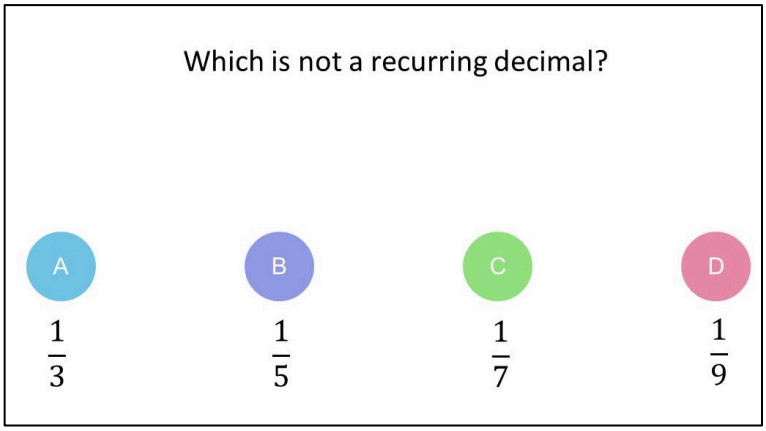
- What do you get when you multiply that number by 100? What do you notice about your answer?
- What do you get when you multiply that number by 1000? What do you notice about your answer?
- When would you multiply by 10, when by 100, when by1000? Why?
- You have 99 times 0.12121… is equal to 12. What does that tell you about 0.12121…?
- Can you simplify that fraction?

Diagnostic Assessment This should take about 5–10 minutes at the end of the lesson.

1. Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, e xplain the right answer or give a remedial task.



B. is the correct answer.

Common Misconceptions

A. Learners are likely to know that one third is 0.3 recurring so they may think the question was 'Which is a recurring decimal' instead of 'Which is NOT…' Learners who give answers **C.** or **D.** probably don't understand how to convert a fraction to a decimal.

You might ask learners what one over nine means and how they would convert this to a decimal. Someone in the class is likely to suggest dividing 9 into 1 (or dividing 1 by 9).

This can't be done right away so ask: 'what is the next step?'
then ask 'what happens when you divide 10 by 9?'
then ask 'what is the remainder?' then say 'try this for yourselves'.
what happens if you go on dividing to find the decimal for 1/9?' https://diagnosticquestions.com

Follow Up

Learners should first do the activity:
Divide Divide https://aiminghigh.aimssec.ac.za/years-7-9-divide-divide/

Then, as suggested in the NEXT section, ask pairs of learners to make up their own tables of 6 fractions paired with 6 equivalent decimals, but jumbled up. Then get other pairs, or the whole class, to solve the problems set by other learners.

The older students and high flyers might be able to use the method to discover that:

$15.6121212\ldots = \frac{15456}{990} = \frac{2576}{165} = 15\frac{101}{165}$ and to make up similar examples for each other.