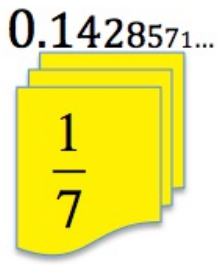
DIVIDE DIVIDE



Fractions, decimals and percentages are three different ways of writing the same number and it is important that we can convert one form into another.

Divide 2 by 11 to get the decimal fraction equivalent to the common fraction two elevenths $\frac{2}{11}$. Carry on dividing until you see a pattern.

Will this pattern continue? Why or why not?

Now use the same method of dividing the numerator by the denominator to find the recurring decimals for all the fractions with 11 as denominator $\frac{1}{11}$, $\frac{3}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{9}{11}$ and $\frac{10}{11}$.

Do you see a pattern? Can you explain why this pattern occurs?

Divide 1 by 4, then divide 2 by 4 and then divide 3 by 4 to get the decimal fractions involving quarters.

Then divide 1 by 5, then divide 2 by 5, then divide 3 by 5 and then divide 4 by 5 to get the decimal fractions involving fifths.

What is the different about these decimals from the decimals involving elevenths?

Divide 1 by 7 to get the decimal fraction equivalent to the common fraction $\frac{1}{7}$.

Carry on dividing until you get 8 places of decimals.

Do you see a pattern? Will this pattern continue? Why or why not?

Now do the same to find the recurring decimals for $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$.

Do you see a pattern? Can you explain why this pattern occurs?

HELP

Think about sharing something with two friends so you get equal shares. You must divide by 3.
The decimal fraction for one third $\frac{1}{3}$ is 0.33333333…
What do you notice about this decimal fraction?

Before starting to divide by 11 and by 7 learners you could write out the multiples of 11:
11, 22, 33, 44, 55, 66, 77, 88, 99 … and the multiples of 7:
7, 14, 21, 28, 35, 42, 49, 56, 63, 70,…
Refer to these lists of multiples as you do the divisions.

NEXT

Investigate the decimal for $\frac{4}{33}$

What is 0.7777… as a fraction?
See https://aiminghigh.aimssec.ac.za/years-7-10-repetition/

GUIDE FOR PARENTS

SOLUTION

$\frac{2}{11} = 0.18181818$ ... This is the recurring decimal with the 2 digits repeating, that is 18… repeating again and again indefinitely.

$\frac{1}{11} = 0.0909$ … , $\frac{3}{11} = 0.27272$ … , $\frac{4}{11} = 0.36363$ … , $\frac{5}{11} = 0.45454$ … , $\frac{6}{11} = 0.54545$ … ,

$\frac{7}{11} = 0.6363$ … , $\frac{8}{11} = 0.72727$ … , $\frac{9}{11} = 0.86181$ … , $\frac{10}{11} = 0.90909$ …

The recurring decimals for the fractions with 11 as denominator all have 2 digits recurring: 09, 18, 27, 36, 45, 54, 63, 72, 81 and 90, the multiples of 9.

This pattern occurs because they can be found by adding the decimal 0.090909… to itself again and again to get two elevenths, three elevenths etc. which is adding 09 to each pair of digits.

The fractions involving quarters are $\frac{1}{4} = 0.25$, $\frac{2}{4} = \frac{1}{2} = 0.5$, $\frac{3}{4} = 0.75$.

The fractions involving fifths are $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$.

These decimals **terminate** because the remainder 0 occurs in the division process.
They are different from the **recurring decimals** involving elevenths for which the remainder 0 never occurs.

$\frac{1}{7} = 0.14285714$ ... This decimal has 6 digits 142857… repeating again and again indefinitely.
This pattern continues because the remainders after division by 7 are repeated in the **same order** and as soon as the remainder is 1 the cycle starts to repeat itself again.

There are only 6 possible remainders after division by 7 so the cycle cannot be longer than 6 digits.

$\frac{2}{7} = 0.28571428$ … , $\frac{3}{7} = 0.42857142$ … , $\frac{4}{7} = 0.57142857$ … , $\frac{5}{7} = 0.71428571$ … ,
$\frac{6}{7} = 0.85714285$ …

Why do this activity?

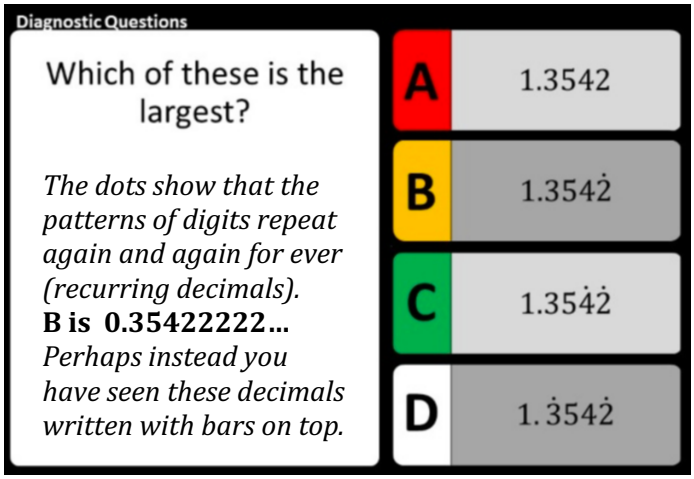
This activity gives learners practice in division of whole numbers and it guides them through the process of converting fractions to decimals. It gives them the experience of looking for patterns, thinking mathematically and explaining why the patterns occur. The class may need to repeat this method many times with other fractions chosen by the teacher if they need more practice in dividing. Each time the teacher should guide their learning by asking searching questions (see Key Questions below) to help them to think mathematically to understand and explain their findings.

The teacher **should not do an example for the learners to show them how**, and then give them similar examples to do. The best way is to **guide learners to do it for themselves** step by step.

Diagnostic Assessment This should take about 5–10 minutes.

Write down this question and ask your children if they have seen dots written above numbers like this.
Some will have seen recurring decimals written with a bar on top instead of dots. For example D can be written with a bar above all the digits 3542. It is helpful for children to realise that some things can be written in more than one way. This quiz is not so much about the way the decimals are written as about what they mean. In particular the question is about which one represents the largest number. **Understanding** is what matters.

Say: "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong but simply thank the learners for giving their answers.
2. It is important for learners to explain the reason for their answer. Try to make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong
3.Then ask them again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

C. is the correct answer.

Common Misconceptions
A. Probably a guess.
B. Does not understand that here the largest decimal must have the largest digit in the 5th place after the decimal point (the 100 thousandths).
D. May just have guessed or think: "The 3 is recurring and it is 3 tenths so it is the largest unit to be recurring".
https://diagnosticquestions.com

Learning objectives

In doing this activity students will have an opportunity to:

- practise the process of division and gain a better understanding of it;
- review and reinforce the understanding of fractions and decimals as two different ways of writing the same number (equivalent forms);
- practise finding the decimal equivalent of a fraction **by division**;
- review and reinforce the understanding of place value;
- develop understanding of the recurring decimals and also of ordering decimals.

Generic competences

In doing this activity students will have an opportunity to develop 'number sense' and 'numeracy' that are important in life generally.

Suggestions for homelearning

To review what the learners know about fractions and decimals start with the Diagnostic Quiz. Then write down some fractions and some decimals. Ask the learners what they are, and what is the same and what is different about them. For example you could use:

$\frac{3}{4}$	0.666…	0.675	$\frac{2}{3}$	0.75	0.8	$\frac{5}{8}$	$\frac{4}{5}$

Learners of age 10 and up should be able to match the pairs of fractions and equivalent decimals and understand that they are different ways of recording the same number. But they do find fractions tricky and much older children and teenagers will benefit from this 'Divide Divide' activity.

Ask: 'Why do you think that division is used to convert fractions to decimals?'
The answer is: **'Fractions are equal shares. The denominator tells you how many parts the whole is split into. The numerator tells you how many shares there are in that fraction'.** Talk about this using examples like $\frac{3}{4}$.

Then tell them to divide 2 by 11 to get the decimal for $\frac{2}{11}$. **Do not allow the use of calculators**. Each learner should work it out individually. If they are not confident about division then ask them to give you the answer to each step and ask them to write it down. If they struggle to do this then try to help, but showing them 'your way' may be more confusing if they have not seen it written your way before.

Ask these questions and listen to the answers given by learners:

'11 is bigger than 2 so what do we do next?'
'Now how many 11s in 20?'
'What is the remainder?'
'Now how many 11s in 90?'
'What is the remainder?'
'How many 11's in 20?'
'What do we do next?'
'Can you see a pattern?'
'Will this pattern continue?'
'Why or why not?'

If learners struggle with the division process then write down the multiples of 1l: they are:
11, 22, 33, 44, 55, 66, 77, 88, 99 … Count together from 1 to 100 clapping on the multiples of 11. Then turn over the paper so nobody can see the list of multiples and skip-count together 11, 22, 33, 44, 55, 66, 77, 88, 99. This skip-counting in multiples is even more important for example when dividing by 7 if children don't know the 7 times table.

Now tell them to divide 3 by 11 to get the decimal for $\frac{3}{11}$. If necessary ask the same sort of questions as you did for dividing 2 by 11 and make sure that the learners do each step for themselves. This will be time consuming but it is damaging when the teacher goes ahead with mathematics to 'cover' the topic and the learners don't understand what they are doing.

Then ask the learners to work out all the decimals for fractions involving quarters and fifths. Check with them that all they can do this successfully. Have a discussion about why some fractions are recurring and others terminate.

It may be possible for your children to work in pairs to find all the decimals for 1, 3, 4, 5, 6, 7 8, 9, and 10 elevenths, working individually and checking the answers they they get with their partners.

Then ask them to work out the decimals for the fractions involving sevenths and have a discussion about the patterns in these decimals.

Key questions

- Show me the denominator in that fraction. What does it tell us about the fraction?
- Show me the numerator in that fraction. What does it tell us about the fraction?
- Why do we divide the numerator by the denominator to convert a fraction into a decimal?
- When you do that division, what remainder do you get?
- What next… how do you use the remainder to continue the division?
- Can you see a pattern? Describe the pattern?
- Why do you think you are getting that pattern?
- Why do you think the digits are repeating themselves?
- Will that repeating pattern stop?
- Why do we call that a recurring decimal?
- Show me the digits that repeat themselves?
- Why do you think the digits repeat themselves?
- How many remainders can you get when you divide by 7?

Follow up

See Repetition https://aiminghigh.aimssec.ac.za/years-7-10-repetition/

Divide Divide is about converting from fractions to decimals using division.
Repetition is about converting from decimals to fractions.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6