

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

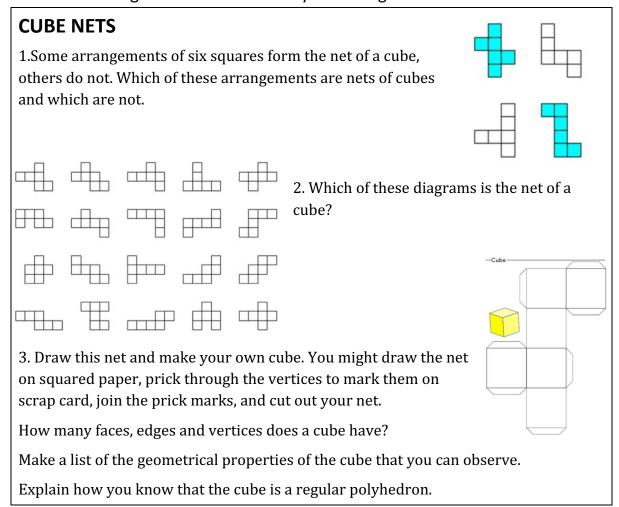
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE

suggests related learning activities for all ages from 4 to 18 on the theme of CUBES

Just choose whatever seems suitable for your group of learners

The original CUBE NETS activity was designed for Years 6 to 10



HELP

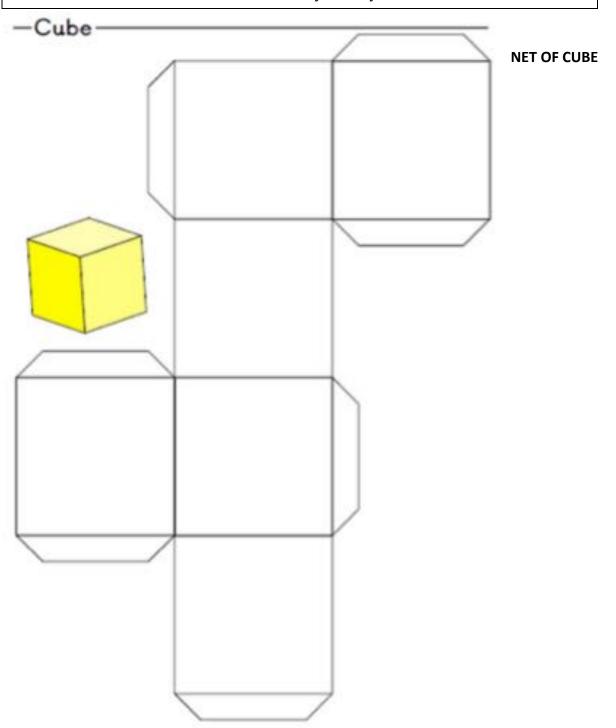
Colouring the faces in 6 different colours makes it much easier to describe the properties of the cube.

NEXT (for older learners)

Draw some sketches of a cube and mark in the planes of reflective symmetry (mirror planes) and the axes of rotational symmetry.

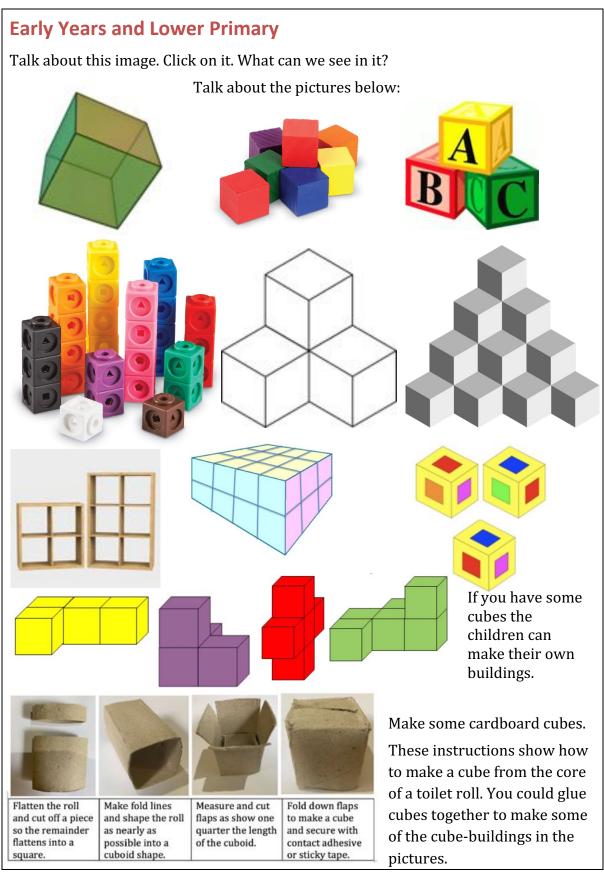
Describe the 9 mirror planes that cut the cube in half so that each half of the cube is a reflection of the other.

Describe the 13 axes of rotational symmetry about which the cube can be rotated into new positions in which it occupies the same space after the rotation as before the rotation. What is the order of rotational symmetry for each axis?



INCLUSION AND HOME LEARNING GUIDE

THEME: CUBES



Upper Primary

You might like to start your lesson by turning a box inside out as shown in the diagram. This will help you to explain what a net is and how the net of a solid can be folded to make the 3D shape. It also helps learners to appreciate how mathematics is used in designing boxes for packing.

You could print the net and ask learners to cut it out and glue it up to make the 3D shape.

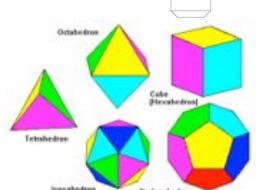
Alternatively draw the net on squared paper and prick through the vertices to mark them on scrap card. The learners can then join the prick marks, cut out the net and make their own cubes.

Each learner can then look at their cube and turn it round in their hands. Ask them what they notice about the cube. Ask how many faces, edges and vertices it has.

Make a list on the board of the geometrical properties of the cube that the learners notice.

Ask the learners to describe the angles of the cube. Ask the learners to measure the edges of the cube.

Tell the learners that we call a solid shape 'regular' when all the angles are equal, and all the edges are equal in length, and so the cube is regular.



HOW TO TURN A BOX INSIDE OUT



Flatten the roll and cut off a piece so the remainder flattens into a square.



Make fold lines and shape the roll as nearly as possible into a cuboid shape.



Measure and cut flaps as show one quarter the length of the cuboid.



Fold down flaps to make a cube and secure with contact adhesive or sticky tape.

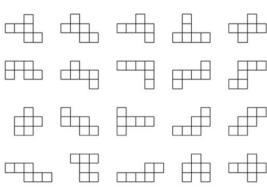
Make some cardboard cubes. These instructions show how to make a cube from the core of a toilet roll. You could glue cubes together to make some of the cube-buildings in the pictures.

Lower Secondary

Start the lesson by turning a packet inside out as described for Upper Primary.

The learners should then make their own cubes. This could be an exercise of accurate geometrical construction. Or learners might use the net on page 2. Or they might draw the net on squared paper, prick through the vertices to mark them on scrap card, join the prick marks, and cut out the net.

Tell the learners to work in pairs and to decide which of these 20 diagrams show nets that can be made into cubes, and which do not. Then have a class discussion about the answers. Ask learners to explain to the class their reasons for deciding one way or the other.



Key questions about the nets:

- Show me the 4 squares that you can make into sides of the cube.
- Show me which square would make the top
- ... and which square would make the bottom of the cube.

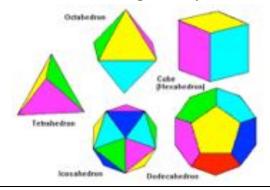
Learners should hold their cubes in their hands, turn them round and look at them from different directions. Ask the learners to make a list of all the properties of a cube that they can think of.

Then in a class discussion get the learners to say what they have noticed and make a list together of all the properties of the cube anyone can think of. Ask the key questions below to prompt them to think of properties.

Key questions

- What shape are the faces?
- How many faces are there altogether?
- What do you notice about the angles?
- How many edges are there altogether? Measure them. How long are they?
- How many vertices are there altogether?
- What do you notice about the vertices?

Tell the learners that we call a solid shape 'regular' when all the angles are equal, and all the edges are equal in length, and so the cube is regular.



Upper Secondary

Make your own cube from scrap card. See instructions on pages 1 and 2.

Decide which of these diagrams are nets of a cube and why.

Answer these questions:

Key questions about the nets:

- Which 4 squares can make into sides of the cube?
- Which square would make the top?
- ... and which square would make the bottom of the cube?
- How many faces are there altogether?
- How many edges are there altogether?
- How many vertices are there altogether?

Key questions about reflections and reflective symmetries:

- Are there any **planes of symmetry**, that is surfaces cutting through the cube that act as mirrors?
- How many planes of symmetry can you find and describe?
- What happens if you cut through the midpoints of 4 edges of the cube parallel to the other faces?
- What happens if you cut through an edge of the cube and through the diagonally opposite edge?

While asking these questions pick up your cube and try to visualise where

Key questions about rotations and rotational symmetries:

Look for the centre of a face. Where is the centre of the opposite face? Hold those 2 points. Now turn the cube round slowly.

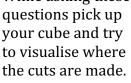
- Can you find any **rotational symmetry**?
- How many times do you turn it?
- Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

Hold your model at two opposite vertices, turn it round slowly and stop when it gets to fill the same position in space. Can you find another rotational symmetry?

- How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

Show me the midpoint of an edge. Now where is the midpoint of the opposite edge? Hold those 2 points. Now turn the cube round slowly. Is there any rotational symmetry?

- How many times do you turn it? Are you turning it through the same angle each time? What is the angle?
- What is the order of that rotational symmetry?

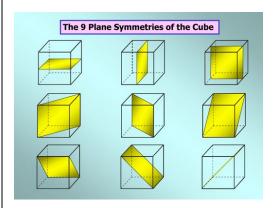


SOLUTION

1. and 2. Thinking of the cube as a box with a lid, you need 4 sides and a top and bottom to make the cube. This can be done from the shaded arrangements in the diagram but not from the unshaded ones.

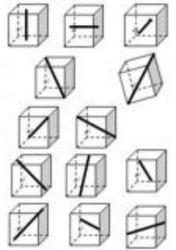
2. A cube has 6 square faces, 12 edges and 8 vertices.

Regular means that all the angles are equal and all the edge lengths are equal. A cube is a regular polyhedron because all its faces are regular squares which are regular polygons.

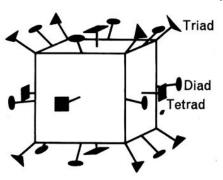


NEXT SOLUTION

This diagram shows the 9 planes of symmetry of the cube. Each plane of symmetry acts as a mirror and in each case one half of the cube is reflected to give the other half as an image after reflection.



The diagram shows the 13 axes of rotational symmetry of the cube. There is the identity transformation.



The 3 lines joining the midpoints of opposite faces (tetrads) are axes of rotational symmetries of order 4. Rotations of 90°, 180° and 270° move the cube into positions

that occupy the same space, 9 rotations in all.

The 4 lines joining opposite vertices (triads) are axes of rotational symmetries of order 3 for which rotations of 120° and 240° move the cube into positions that occupy the same space (8 rotations).

The 6 lines joining midpoints of opposite edges (diads) are axes of rotational symmetries of order 2 for which rotations of 180° move the cube into positions that occupy the same space (6 rotations). This makes 24 rotations including the identity.

Why do this activity?

The cube is the simplest 3D shape, apart perhaps from the tetrahedron, but it has many properties that are easier to recognise than in some of the other 3D shapes. Making their own cubes helps learners to visualise the shape and to understand its properties. This activity gives them practice in making accurate geometrical drawings (measuring right angles and lengths of lines).

Some learners, who find other topics in mathematics more difficult, may be able to do better with this activity because it is concrete and practical. The activity builds on knowledge and understanding of properties of 2D shapes and also connects to work on transformations (reflections and rotations). It paves the way for work on surface area and volume.

Learning objectives

In doing this activity students will have an opportunity to develop their understanding and appreciation of the properties of 3D shapes including reflections and rotations.

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- think mathematically and to reason logically;
- interpret and solve problems in a variety of situations;
- develop the skill of interpreting and creating visual images and to 'think in pictures'.

DIAGNOSTIC ASSESSMENT This can be done before or after the lesson and as a group as described below, or the question can be answered individually. Show this question and say:

> If the net shown is folded to make a cube, which

letter is opposite X?

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".

- 1. Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
- 2. It is important for learners to explain the reason for their answer so that, by putting their

C D thinking into words, they develop communication skills and gain a better understanding.

- 3. With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
- 4. Ask the learners to vote again for the right answer by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

The correct answer is: D

Students who gave any of the answers A, B or C cannot visualise how to fold the net into a cube. https://diagnosticquestions.com

Follow up

Other activities about cubes:

How do you see it? https://aiminghigh.aimssec.ac.za/years-4-to-9-how-do-you-see-it/ Painted cube https://aiminghigh.aimssec.ac.za/grades-7-to-10-painted-cube/

Three Views is about plan and elevation drawings which are important in engineering and architectural design: https://aiminghigh.aimssec.ac.za/years-4-8-three-views/



Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za

Subscribe to the MATHS TOYS YouTube Channel

https://www.youtube.com/c/mathstoys

Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

Lower Primary Upper Primary Lower Secondary **Upper Secondary** Age 11 to 15 Age 15+ Age 8 to 11 Approx. Age 5 to 8 South Africa Grades R and 1 to 3 Grades 7 to 9 Grades 10 to 12 Grades 4 to 6 East Africa Nursery and Primary 1 to 3 Primary 4 to 6 Secondary 1 to 3 Secondary 4 to 6 USA Kindergarten and G1 to 3 Grades 4 to 6 Grades 7 to 9 Grades 10 to 12 UK Reception and Years 1 to 3 Years 7 to 9 Years 4 to 6 Years 10 to 13