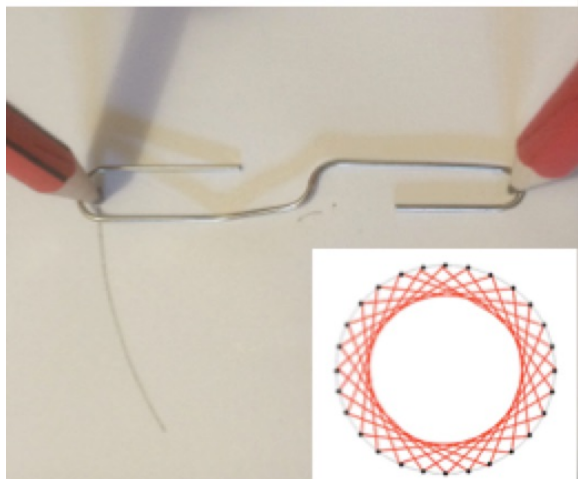# CLOCK ARITHMETIC AND ENVELOPES
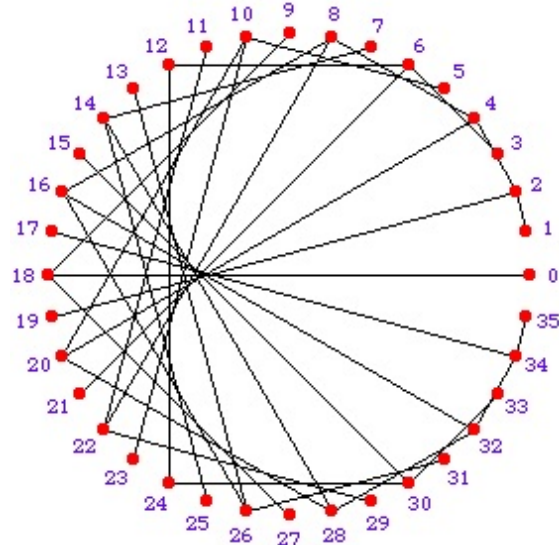
PAPERCLIP COMPASS

Double click on the picture to start the movie showing how to draw a circle using the paperclip compass.

Or go to: https://youtu.be/iewxclQEAnk

Cardioid technology is used in microphones and speakers. Cardioids appear in the motion of planets and in many other ways. See this YouTube video. www.youtube.com/watch?v=tTYGLoVb5xA

To draw a cardioid, first draw a circle and carefully mark 36 points around the circle at 10 degree intervals and number the points n = 0 to 35. You are going to draw cardioids and other envelopes by joining each point n to the point given by some function of n.

1. Join n →2n using clock arithmetic.

You already use clock arithmetic in:
12s with months and with hours;
in 60s with minutes and seconds;
 also in 7's with days of the week.

Double each number and join it to the point given by the remainder after dividing by 36 (clock arithmetic). For example 19 is joined to 2 because 2 times 19 is 38 which is 1× 36 + 2. Similarly 25 is joined to 14 because 2 × 25 is 50 which is 1× 36 + 14.

2. This shape is called a nephroid. Start with a base circle and a diameter of that circle. Construct the nephroid by drawing circles with centres on the base circle and each tangent to the diameter of the base circle.

The cardioid and nephroid are examples of envelopes which are curves that are tangent to a family of curves at each point of contact.

3. Experiment with different patterns using Method 1 and other functions, for example n →3n and n → 2n + 5 etc.
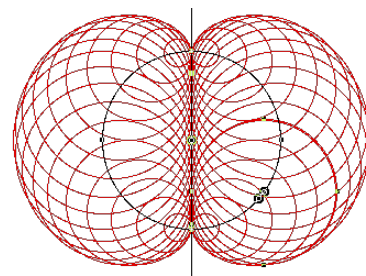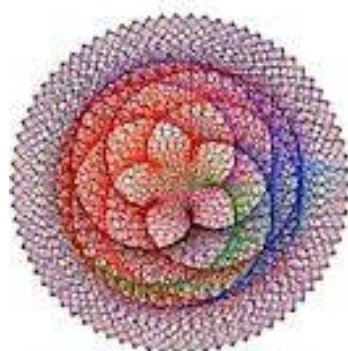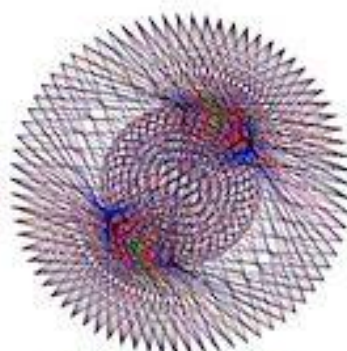
THE DANCE OF THE PLANETS

If you join the positions of pairs of planets in our Solar System at different times you get these patterns.

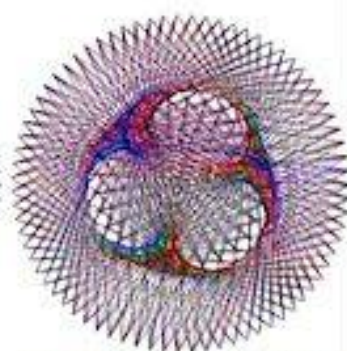This happens because the motion of the planets around the sun is periodic.

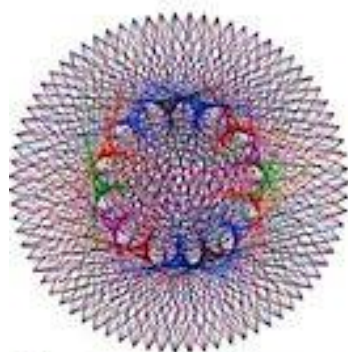The video (link above) shows the pattern for the Earth and Venus.
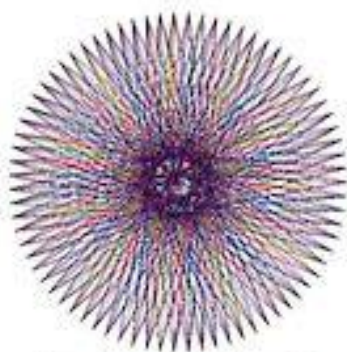
Earth & Venus
8 Orbits
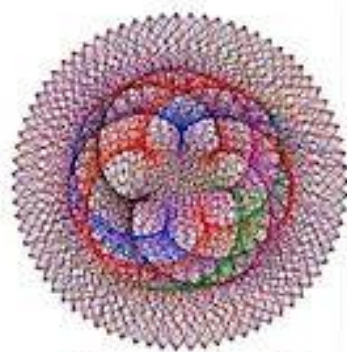
Mars & Venus
7 Orbits
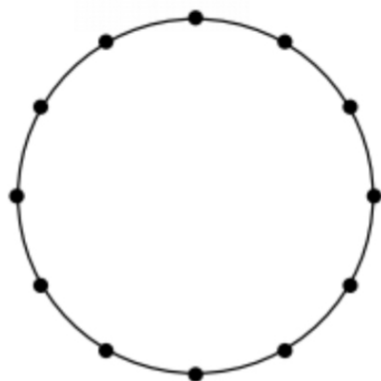
Saturn & Jupiter
7 Orbits

Uranus & Saturn
7 Orbits

Jupiter & Earth
7 Orbits

Mars & Earth
8 Orbits

Help

If you are struggling to measure angles and mark points on a circle you could start with this circle with 12 points already marked and join each point n to $2n$ where, like a clock, the point 10 is joined to 8 because $2 \times 10 = 20 = 12 + 8$

Then try 6 points or 24 points (see pages 5 and 6).

When you succeed in drawing patterns with a few points you might find it easier to copy one of the designs rather than working from a formula.

Extension

Mystic Rose
https://aiminghigh.aimssec.ac.za/years-7-12-mystic-rose/

A different construction – Join every point to every other point around the circle. How many lines do you need to draw?