BULDING FUNCTIONS

[−2] [× 3] [÷2] [+5]

Amy and her friends have built some functions. If they all input the number 1 into their functions Amy's output would be 6, Busi's 1, Chris's 6½ and Dudu's 3½.

Amy's function a → [+5] → $a+5$

Busi's function b → [×3] → [-2] → $3b-2$

Chris's function c → [÷2] → [×3] → [+5] → $3c/2 + 5$

Dudu's function d → [-2] → [×3] → [÷2] → [+5] → $3(d-2)/2 + 5 = 3d/2 + 2$

Build some of your own functions using the operators [subtract 2], [multiply by 3], [divide by 2] and [add 5].

Choose inputs and give the corresponding outputs to show how your functions work.

Your inputs can be numbers or variables. Make as many different functions as you can using 2 of the 4 operations without repetition.

Click here for a poster that you can fill in to show all the different functions that can made by combining 2 of these 4 operations without repetition. In the first column put the functions that have [subtract 2] first. An example has been shown in the table.

HELP

Start with just 2 operators and numerical inputs. Choose as many different pairs of operators as you can find. Try to find all the possibilities.

Record your functions like Busi but with numbers for input and output.

NEXT

Make as many functions as you can that combine 3 of the 4 operations without repetition.

Then find the functions that combine all 4 operations without repetition.

Work with a friend and make your own functions. Then try to work out the rule made up by each other. If you give your friend the your function rule and an output can he work out the input?

NOTES FOR TEACHERS

Solution

There are 12 functions using 2 of the 4 given operations

−2 first	×3 first	÷2 first	+5 first
6 → 4 → 12 $\boldsymbol{x} \to \boldsymbol{x-2} \to 3(x-2)$	6 → 18 → 16 $\boldsymbol{x} \to \boldsymbol{3x} \to 3x-2$	6 → 3 → 1 $\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x-2$	6 → 11 → 9 $\boldsymbol{x} \to \boldsymbol{x+5} \to 3(x+5)$
6 → 4 → 2 $\boldsymbol{x} \to \boldsymbol{x-2} \to \frac{1}{2}(x-2)$	6 → 18 → 9 $\boldsymbol{x} \to \boldsymbol{3x} \to \frac{3}{2}x$	6 → 3 → 9 $\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{3}{2}x$	6 → 11 → 33 $\boldsymbol{x} \to \boldsymbol{x+5} \to 3(x+5)$
6 → 4 → 9 $\boldsymbol{x} \to \boldsymbol{x-2} \to x+3$	6 → 18 → 23 $\boldsymbol{x} \to \boldsymbol{3x} \to 3x+5$	6 → 3 → 8 $\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x+5$	6 → 11 → $\frac{11}{2}$ $\boldsymbol{x} \to \boldsymbol{x+5} \to \frac{1}{2}(x+5)$

There are 24 functions using all 4 operations. Perhaps only a few learners will find them all but the complete table is given here for reference.

$\boldsymbol{x} \to \boldsymbol{x-2} \to 3x-2$ $\to \frac{1}{2}(3x-2) \to \frac{1}{2}(3x+4)$ -2, x3, ÷2, +5 6 -> 11	$\boldsymbol{x} \to \boldsymbol{3x} \to 3x-2$ $\to \frac{1}{2}(3x-2) \to \frac{1}{2}(3x+8)$ x3, -2, ÷2, +5 6 ->13	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x-2$ $\to \frac{3}{2}x-6 \to \frac{3}{2}x+1$ ÷2,-2,x3,+5 6 -> 10	$\boldsymbol{x} \to \boldsymbol{x+5} \to x+3$ $\to 3x+9 \to \frac{3}{2}(x+3)$ +5,-2,x3,÷2 6 -> 13½
$\boldsymbol{x} \to \boldsymbol{x-2} \to 3(x-2)$ $\to 3x+1 \to \frac{1}{2}(3x+1)$ -2, x3, +5, ÷2 6 -> 9½	$\boldsymbol{x} \to \boldsymbol{3x} \to 3x-2$ $\to 3x+3 \to \frac{1}{2}(3x+3)$ x3,-2,+5,÷2 6 -> 10½	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x-2$ $\to \frac{1}{2}x+3 \to \frac{3}{2}x+9$ ÷2,-2,+5,x3 6 -> 18	$\boldsymbol{x} \to \boldsymbol{x+5} \to x+3$ $\to \frac{1}{2}(x+3) \to \frac{3}{2}(x+3)$ +5,-2,÷2,x3 6 -> 13½
$\boldsymbol{x} \to \boldsymbol{x-2} \to \frac{1}{2}(x-2)$ $\to \frac{3}{2}(x-2) \to \frac{3}{2}x+2$ -2, ÷2, x3, +5 6 -> 11	$\boldsymbol{x} \to \boldsymbol{3x} \to \frac{3}{2}x$ $\to \frac{3}{2}x-2 \to \frac{3}{2}x+3$ x3,÷2,-2,+5 6 -> 12	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{3}{2}x$ $\to \frac{3}{2}x-2 \to \frac{3}{2}x+3$ ÷2,x3,-2,+5 6 -> 12	$\boldsymbol{x} \to \boldsymbol{x+5} \to 3(x+5)$ $\to 3x+13 \to \frac{1}{2}(3x+13)$ +5,x3,-2,÷2 6 -> 15½
$\boldsymbol{x} \to \boldsymbol{x-2} \to \frac{1}{2}(x-2)$ $\to \frac{1}{2}x+4 \to \frac{3}{2}x+12$ -2, ÷2, +5, x3 6 -> 21	$\boldsymbol{x} \to \boldsymbol{3x} \to \frac{3}{2}x$ $\to \frac{3}{2}x+5 \to \frac{3}{2}x+3$ x3,÷2,+5,-2 6 -> 12	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{3}{2}x$ $\to \frac{3}{2}x+5 \to \frac{3}{2}x+3$ ÷2,x3,+5,-2 6 -> 12	$\boldsymbol{x} \to \boldsymbol{x+5} \to 3(x+5)$ $\to \frac{3}{2}(x+5) \to \frac{3}{2}x+\frac{11}{2}$ +5,x3,÷2,-2 6 -> 14½
$\boldsymbol{x} \to \boldsymbol{x-2} \to x+3$ $\to 3(x+3) \to \frac{1}{2}(3x+9)$ -2, +5, x3, ÷2 6 -> 13½	$\boldsymbol{x} \to \boldsymbol{3x} \to 3x+5$ $\to 3x+3 \to \frac{3}{2}(x+1)$ x3,+5,-2,÷2 6 -> 10½	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x+5$ $\to \frac{1}{2}x+3 \to \frac{3}{2}x+9$ ÷2,+5,-2,x3 6 -> 18	$\boldsymbol{x} \to \boldsymbol{x+5} \to \frac{1}{2}(x+5)$ $\to \frac{1}{2}(x+1) \to \frac{3}{2}(x+1)$ +5,÷2,-2,x3 6 -> 10½
$\boldsymbol{x} \to \boldsymbol{x-2} \to x+3$ $\to \frac{1}{2}(x+3) \to \frac{3}{2}(x+3)$ -2, +5, ÷2, x3 6 -> 13½	$\boldsymbol{x} \to \boldsymbol{3x} \to 3x+5$ $\to \frac{1}{2}(3x+5) \to \frac{1}{2}(3x+1)$ x3,+5,÷2,-2 6 -> 9½	$\boldsymbol{x} \to \boldsymbol{\frac{1}{2}x} \to \frac{1}{2}x+5$ $\to \frac{3}{2}x+15 \to \frac{3}{2}x+13$ ÷2,+5,x3,-2 6 -> 22	$\boldsymbol{x} \to \boldsymbol{x+5} \to \frac{1}{2}(x+5)$ $\to \frac{3}{2}(x+5) \to \frac{1}{2}(3x+11)$ +5,÷2,x3,-2 6 -> 14½

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

The input to this function mapping is 10. What is the output?

10 → -2 → $\times 3$ → $\div 2$ → $+5$ →

A. 31 B. 17 C. 20½ D. 18

2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. It is important for learners to explain the reason for their answer because they develop their communication skills doing so and als otherwise many learners will just make a guess.
4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
   The correct answer is **B:**
   - **A.** Learners may have added not divided by 2.
   - **C.** Learners may have worked from right to left.
   - **D.** Learners may have miscalculated or guessed.

Why do this activity?

This is an open-ended activity to accommodate all learners in a class so all learners will get practice in combining operators to get different functions. Some learners can be encouraged to find the algebraic rules for the functions and some learners can work entirely with numbers applying the operators in different orders. Learners can be invited to choose their own challenges. The teacher can decide to challenge the class between them to find a complete set of functions to give the learners experience of organising the search systematically to fill in a poster.

Learning objectives

In doing this activity students will have an opportunity to:

- investigate composition of functions;
- experience the advantage of working systematically to find all possible cases;
- meet permutations and combinations informally;
- make sense of mapping diagrams and their use in illustrating composite functions.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations;

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;

Suggestions for teaching

-2	$\times 3$	$\div 2$	$+5$

Draw these 4 boxes on the board and tell your class they are going to build their own functions using these operators.

Then put Amy's rule on the board and ask the learners to give the outputs for different inputs. Depending on the class you can use numbers as inputs or both numbers and variables. Then in the same way discuss one or more of the other 3 examples.

Then tell your class to make up some of their own functions by putting together some of the operators [subtract 2], [multiply by 3], [divide by 2] and [add 5]. Tell them to choose an input and give the corresponding output to show how their functions work and that they can use numerical inputs or variables.

If you wish, get your learners to work in groups and tell them to decide in their groups to find as many different functions as possible made up of two operators or tell them they can work with 3 or 4 operators.

Later you can have a class discussion and record on the board or on a poster the functions found by the learners. You might like to discuss with the class why it makes no difference in which order they use the operations -2 and +5 if they take them one after the other, or the operations ×3 and ÷2, but a change in the order gives different outputs for other pairs of operations.

In order to go through all the possibilities systematically you might like to use a table to record the 12 function rules that use 2 of the 4 given operations exactly as given above. Perhaps you could **make a poster** to go on the classroom wall. If the whole table is not filled in during the lesson learners can be asked to find the missing ones and fill them in as they are found.

If you decide to collect all the functions made up of 4 operators then, as the table fills up, you can ask "has anyone got a function to go here?" indicating one of the empty boxes. Perhaps make a poster to go on the classroom wall and enter new functions as they are found.

Always reward learners with praise for finding an answer that nobody else has found. Help learners to understand that determination and hard work are needed for success, not cleverness. Never say 'how clever of you' or 'that was clever (or smart)' but rather praise their persistence or for having kept on trying.

Key questions

- Pick two operations, which will you do first?

- What happens to the input (suggest a number or a letter) if you do those two operations?
- Does it make a difference if you do this operation before that or change the order? Why or why not?
- What happens if your input is the variable x?
- Can you find any more functions using those operations in different orders?

Follow up

These can be done before the Building Functions activity:

https://aiminghigh.aimssec.ac.za/mind-reader/

https://aiminghigh.aimssec.ac.za/shifting-times-tables/

https://aiminghigh.aimssec.ac.za/function-game/

A similar activity: https://aiminghigh.aimssec.ac.za/function-flow/

Inverse functions: https://aiminghigh.aimssec.ac.za/swop/

https://aiminghigh.aimssec.ac.za/undoing/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6