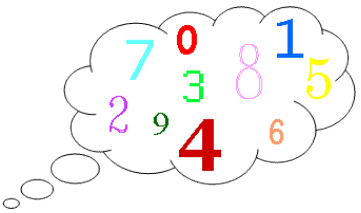
THINK OF TWO WHOLE NUMBERS

Here is an alternative and more unusual version of the "Think of a Number" trick that you may have heard of before.



Think of two whole numbers under 10.
Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.

From your answer I can work out both your numbers very quickly.

How?

Choose some different pairs of numbers and repeat the process.

Can you find a good explanation of how the trick works?

HELP

Try this with different pairs of numbers and write down the answers you get at every step.

Think about the 2 numbers and the final answer each time.

What do you notice?

That could be the key to how the trick works.

NEXT

Make up a puzzle of your own involving 2 numbers.

Try your trick on other people. Can you astonish them with your mathemagic?

Challenge them to find out how it works if they say it is a trick.

NOTES FOR TEACHERS

SOLUTION

Here is the working for one pair of numbers x and y.

You tell me 44 and I can work out both the numbers very quickly.

Because $10x + y + 9 = 44$, I just subtract 9 from 44. (44 – 9 = 35)

So the numbers are 3 and 5.

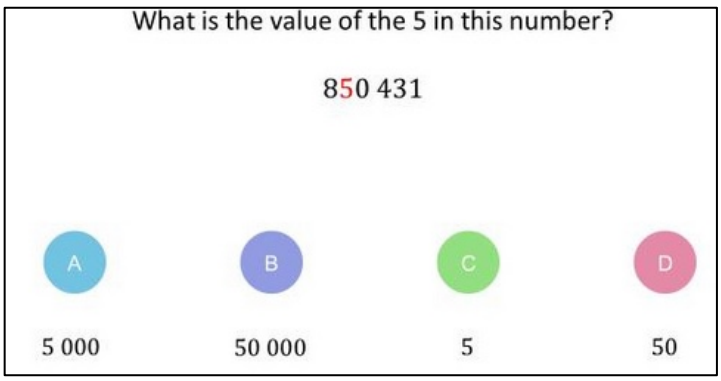
This works for any pair of numbers.

Think of two whole numbers under 10.	x and y	3 and 5
Take one of them and add 1.	$x + 1$	4
Multiply by 5.	$5(x + 1) = 5x + 5$	20
Add 1 again.	$5x + 6$	21
Double your answer.	$10x + 12$	42
Subtract 1.	$10x + 11$	41
Add your second number.	$10x + y + 11$	46
Add 2.	$10x + y + 13$	48
Double again.	$20x + 2y + 26$	96
Subtract 8.	$20x + 2y + 18$	88
Halve this number and tell me your answer.	$10x + y + 9$	44

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 for D".



1.Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2.It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. Explain the right answer or give a remedial task.

The correct answer is **B.**

Learners choosing other answers do not understand place value. https://diagnosticquestions.com

Why do this activity?

This is an intriguing kind of "puzzle-game" that learners can take from the maths classroom into the playground or use to or impress the adults that they know. It can lead into recording the instructions as a list of algebraic expressions and then using the final expression to explain the method used to guess the two numbers. This revises and builds on what the learners already know about place value and leads to more work on algebra.

Learning objectives

In doing this activity students will have an opportunity to:

- reinforce their understanding of place value;
- develop their ability to notice number patterns;
- prepare to use algebra as a tool in problem solving.

Generic competences

In doing this activity students will have an opportunity to:

- think mathematically, reason logically and give explanations;
- interpret and solve problems;
- work in a team, collaborate/work with a partner or group
- communicate:
    - exchange ideas, criticize, and present information and ideas to others
    - analyze, reason and record ideas effectively.

Suggestions for teaching

Start with formative assessment using the Diagnostic Quiz and try to make sure that all the learners understand place value. This will give a clue that the learners may need to use place value in the lesson but don't go out of your way to tell them that.

Make a table on the board listing the instructions. You could then introduce the problem by giving one learner at the front a sheet of paper and a thick coloured pen, then turning your back to the class and asking this learner to write 2 whole numbers under 10 on the paper and to show the class but not to let you see it.

Then go through the instructions slowly, one step at a time, asking the class to give you the answers they get (that way they check each others' calculations) and write the answers on the board in a column (**not** the algebra at this stage, just the numbers). At the end tell the learners their two numbers. Repeat this several times leaving the columns of calculations and answers on the board. Can anyone explain how it's done?

Learners could then work individually to give them 'thinking time', then work in pairs to support each other and to give an opportunity for mathematical talk, and finally there could be a class discussion.

In the concluding plenary ask learners to share any insights and strategies that helped them succeed at this task. If nobody in the class can explain it then tell them that you will give them the time until their next maths lesson to discover the trick.

If the learners discover and can explain the trick in the same lesson, or if not in the next lesson, repeat the trick with numbers once, then slowly with letters and ask the learners to tell you what you should write, perhaps using First and Second to refer to the numbers instead of letters *x* and *y*.

If the learners have not met algebra before then letters can be introduced as a 'shorthand' If they have met algebra, take sufficient time over this for the learners to follow the algebra step by step. Try to get the learners to suggest the algebraic expressions rather than telling the class what they should be.

Recording the process algebraically will review their algebra and help them to give a clear explanation for why the trick works.

Key questions

- Have you tried with several pairs of numbers to see what is happening?
- What can you say about the answer and the first number that was chosen?
- What can you say about the answer and the second number that was chosen?
- Have you tried doing it with someone else whose numbers you do not know?
- Have you tried using two letters in place of the two numbers?

Follow up

Learners could go on to Magic 13837
https://aiminghigh.aimssec.ac.za/years-6-10-magic-13837/
Which Scripts https://aiminghigh.aimssec.ac.za/years-6-7-which-scripts/
or Multiply the Addition Square NRICH 2922.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13