



## SYMMETRICALITY

Add up these 3 equations. What do you notice?

$$y + z = 2$$

$$x + z = 1$$

$$x + y = 9$$

Can you find  $x$ ,  $y$  and  $z$ ?

Add up all 5 equations given below. What do you notice?

$$b + c + d + e = 4$$

$$a + c + d + e = 5$$

$$a + b + d + e = 1$$

$$a + b + c + e = 2$$

$$a + b + c + d = 0$$

Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$ .

## HELP

If you know the values of  $x + y$  and of  $x + y + z$  can you find the value of  $z$ ?

## NEXT

Use a similar (but importantly different) method to solve the following:

$$xy = 1$$

$$yz = 4$$

$$zx = 9.$$

## NOTES FOR TEACHERS

### SOLUTION

Adding the first 3 equations we get  $2x + 2y + 2z = 12$  so  $x + y + z = 6$ .

As  $y + z = 2$  this gives  $x + 2 = 6$  so  $x = 4$

As  $x + z = 1$  this gives us  $y + 1 = 6$  so  $y = 5$

As  $x + y = 9$  this gives  $z + 9 = 6$  so  $z = -3$

Check:  $x + y + z = 4 + 5 + (-3) = 6$ .

Adding the 5 equations we get  $4a + 4b + 4c + 4d + 4e = 12$ .

So  $a + b + c + d + e = 3$ .

Notice that each equation has just one variable missing.

The first equation gives  $4 + a = 3$  so  $a = -1$

The second equation gives  $5 + b = 3$  so  $b = -2$

The third equation gives  $1 + c = 3$  so  $c = 2$

The fourth equation gives  $2 + d = 3$  so  $d = 1$

The fifth equation gives  $0 + e = 3$  so  $e = 3$

Check  $a + b + c + d + e = (-1) + (-2) + 2 + 1 + 3 = 3$

### Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:

**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**

Solve the following pair of simultaneous equations:

$$\begin{aligned}4x + y &= 13 \\3x - y &= 8\end{aligned}$$



$$\begin{aligned}x &= 1, \\y &= 3\end{aligned}$$



$$\begin{aligned}x &= 3, \\y &= 1\end{aligned}$$



$$\begin{aligned}x &= 3, \\y &= 10\end{aligned}$$



$$\begin{aligned}x &= 5, \\y &= -7\end{aligned}$$

2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

#### The correct answer is B

- A. The student has written the solutions the wrong way around.
- B. Correct solution
- C. The student has calculated x correctly, but then used x rather than 4x in the first equation to solve for y.
- D. The student has subtracted the equations, rather than adding, to find x.

<https://diagnosticquestions.com>

## Why do this activity?

While this activity involves solving 3 simultaneous equations, and then 5 simultaneous equations, it is simple enough for learners to work out the solution because all the coefficients are one and the numbers involved are all small. Learners should be told to look for patterns. They will get practice at working algebraically and can be congratulated on their success and told that they have done extremely well because the tests they get will only involve 2 equations in 2 unknowns. The activity enables teachers to plan for different abilities by giving only the set of 3 equations to weaker learners and only giving the set of 5 equations to learners when they have successfully solved the set of 3 equations.

## Learning objectives

In doing this activity students will have an opportunity to think flexibly about solving simultaneous equations and to deepen their understanding of the methods.

## Generic competences

In doing this activity students will have an opportunity to **think flexibly**, be creative and innovative and apply knowledge and skills;

## Suggestions for teaching

Give the learners the set of 3 equations and ask them to work in pairs to find the solutions. Most learners will probably use trial and improvement but let them carry on and when they get the answers ask them if they can find another method. If you observe learners working algebraically, and getting the solution successfully, congratulate them quietly and give them the set of 5 equations.

When most learners have had some success, either by trial and improvement or algebraically, ask pairs of learners to come to the board and explain what they have done to the class. Include a pair who have found the solutions using algebra. If most of the learners have solved both sets of equations then ask a pair who have solved the set of 5 equations to explain what they have done to the class. Then go over this method yourself writing it up carefully on the board.

## Key questions

- What patterns do you see?
- What can you say about the sum of all the unknowns? ( $x + y + z$ ) or ( $a + b + c + d + e$ )
- Which unknown is missing from that equation?
- Can you use that to find the value of the missing unknown?

## Follow up

If learners solve both of the above sets of equations you could ask them to use a similar (but importantly different) method to solve the following:

$$\begin{aligned}xy &= 1 \\yz &= 4 \\zx &= 9.\end{aligned}$$

(Note they should be able to find out that  $xyz=6$ )

Polycircles <https://aiminghigh.aimssec.ac.za/years-11-12-polycircles/>

Matchless <https://aiminghigh.aimssec.ac.za/years-9-12-matchless-6/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6